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A
TREATISE
OF
ARITHMETIC.

BEING

A plain and familiar method, ſuitable to the meaneſt capacity, for the full underſtanding of that incomparable ART.

BY

EDWARD COCKER.

A NEW EDITION,

Reviſed and corrected by JOHN MAIR, A. M.

EDINBURGH:
Printed by A. DONALDSON and J. REID,
For ALEXANDER DONALDSON.
MDCCLXII.

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COCKER's VULGAR ARITHMETIC, though in high eſteem for many years after its firſt publication, and though a book ſtill much called for, has nevertheleſs had the misfortune, for a long time paſt, to have been very careleſsly printed; all the modern impreſſions, without exception, ſo much abounding in omiſſions, falſe figures, erroneous anſwers, and typographical blunders of every ſort, that the book, in many places, tends to miſlead or puzzle, rather than inſtruct the learner. And it being the deſign of the preſent editor to remedy this evil, I have, in compliance with his repeated intreaties, and taking for my guide the moſt ancient editions, particularly that publiſhed at London 1697, endeavoured to ſupply defects, and rectify miſtakes of every kind, in order to reſtore the book to its primitive purity, and once more furniſh the public with a genuine and correct copy of this part of Mr COCKER's works. —— That it may anſwer the end propoſed by the editor, and prove ſerviceable to the reader, is the earneſt deſire of him who wiſheth the propagation of every uſeful art and ſcience.

Auguſt 1. 1751.

JOHN MAIR

To his Much Honoured Friends,

MANWARING DAVIES, of the Inner Temple, Eſq;

AND

Mr HUMPHREY DAVIES, of St Mary, Newington Butts, in the county of Surry,

John Hawkins, as an acknowledgment of unmerited favours, humbly dedicateth this Manual of Arithmetic.

To the READER.

Courteous Reader,

I *Having had the happineſs of an intimate acquaintance with Mr Cocker in his lifetime, often ſolicited him to remember his promiſe to the world, of publiſhing his arithmetic; but (for reaſons beſt known to himſelf) he refuſed it; and (after his death) the copy falling accidentally into my hands, I thought it not convenient to ſmother a work of ſo conſiderable a moment; not queſtioning but it might be as kindly accepted, as if it had been preſented by his own hand. The method is familiar and eaſy, diſcovering as well the theoric as the practice of that neceſſary art of vulgar arithmetic: and in this new edition there are many remarkable alterations, for the benefit of the teacher or learner, which I hope will be very acceptable to the world. I have alſo performed my promiſe in publiſhing the decimal arithmetic, which finds encouragement to my expectation and the bookſellers too. —— I am,*

Thine to ſerve thee,

John Hawkins.

Courteous Reader,

B*Eing well acquainted with the deceaſed author, and finding him knowing and ſtudious in the myſteries of numbers and algebra, of which he had ſome choice manuſcripts, and a great collection of printed authors in ſeveral languages, I doubt not but he hath writ his arithmetic ſuitable to his own preface, and worthy acceptation; which I thought fit to certify on a requeſt to that purpoſe made to him that wiſheth thy welfare, and the progreſs of arts.*

November 27.
1677.

John Collins.

This manual of arithmetic is recommended to the world by us whoſe names are ſubſcribed, viz.

Mr
Jo. Collens, } Mat.
Ja. Atkinſon, } Mat.
Peter Perkins, } Mat.
Rich. Lawrence, ſen.
Eleazar Wigan

Mr
Rich. Noble of Guilford
William Norgate
William Maſon
Steph. Thomas
Peter Storey

Mr
Benj. Tichbourn
Joſeph Symmonds
Jerem. Milles
Joſiah Cuffly
John Hawkins

And generally approved by all ingenious artiſts.

THE AUTHOR's

PREFACE.

BY the ſacred influence of divine providence, I have been inſtrumental to the benefit of many, by virtue of thoſe uſeful arts, writing and ingraving; and do now, with the ſame wonted alacrity, caſt this my arithmetical mite into the public treaſury, beſeeching the Almighty to grant the like bleſſing to theſe as to my former labours.

Seven ſciences, ſupremely excellent,
Are the chief ſtars in wiſdom's firmament;
Whereof ARITHMETIC is one, whoſe worth
The beams of profit and delight ſhine forth:
This crowns the reſt, this makes man's mind complete;
This treats of numbers, and of this we treat.

I have been often deſired by my intimate friends to publiſh ſomething on this ſubject; who in a pleaſing freedom have ſignified to me, that they expected it would be extraordinary. How far I have anſwered their expectation, I know not: but this I know, that I have deſigned this work not extraordinary abſtruſe or profound; but have, by all means poſſible within the circumference of my capacity, endeavoured to render it extraordinary uſeful to all thoſe whoſe occaſions ſhall induce them to make uſe of numbers. If it be objected, That the books already publiſhed, treating of numbers, are innumerable; I anſwer, That it is but a ſmall wonder, ſince the art is infinite. But that there ſhould be ſo many excellent tracts of practical arithmetic extant, and ſo little practiſed, is to me a greater wonder; knowing, that as merchandiſe is the life of the weal-public, ſo practical arithmetic is the ſoul of merchandiſe. Therefore I do ingenuouſly profeſs, that, in the beginning of this undertaking, the numerous concerns of the honoured merchant firſt poſſeſſed my conſideration:

and how far I have accommodated this compoſure for his moſt worthy ſervice, let his own profitable experience be judge.

Secondly, For your ſervice, moſt excellent profeſſors, whoſe underſtandings ſoar to the ſublimity of the theory and practice of this noble ſcience, was this arithmetical tractate compoſed; which you may pleaſe to employ as a monitor to inſtruct your young tyroes, and thereby take occaſion to reſerve your precious moments, which might be exhauſted that way, for your more important affairs.

Thirdly, For you, the ingenious offspring of happy parents, who will willingly pay the full price of induſtry and exerciſe for thoſe arts and choice accompliſhments, which may contribute to the felicity of your future ſtate; for you, I ſay, ingenious practitioners, was this work compoſed, which may prove the pleaſure of your youth, and the glory of your age.

Laſtly, For you, the pretended numeriſts of this vapouring age, who are more diſingeniouſly witty to propound unneceſſary queſtions, than ingeniouſly judicious to reſolve ſuch as are neceſſary; for you was this book compoſed and publiſhed: if you will deny yourſelves ſo much as not to invert the ſtreams of your ingenuity, but, by ſtudiouſly conferring with the notes, names, orders, progreſs, ſpecies, properties, proprieties, proportions, powers, affections, and applications of numbers delivered herein, become ſuch artiſts indeed, as you now only ſeem to be. This arithmetic, ingeniouſly obſerved, and diligently practiſed, will turn to good account to all that ſhall be concerned in accounts; all whoſe rules are grounded on verity, and delivered with ſincerity. The examples are built up gradually from the ſmalleſt conſideration to the greateſt. All the problems or propoſitions are well weighed, pertinent, and clear; and not one of them throughout the tract taken upon truſt. Therefore now,

Zoilus and Momus, lie you down and die;
For theſe inventions your whole force defy.

EDWARD COCKER.

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ARITHMETIC.

CHAP. I.

Notation of Numbers.

ARITHMETIC is the art of numbering, or knowledge which teacheth to number well, *viz.* the doctrine of accounting by numbers. And there are divers ſpecies and kinds of arithmetic and geometry; the which we do intend to treat of in order; applying the principles of the one to the definitions of the other. For as magnitude or greatneſs is the ſubject of geometry, ſo multitude or number is the ſubject of arithmetic; and if ſo, then their firſt principles and chief fundamentals muſt have like definitions, or at leaſt ſome congruency.

2. Number is that by which the quantity of any thing is expreſſed or numbered: as, the unit is the number by which the quantity of one thing is expreſſed, or ſaid to be *one;* and two by which it is named *two;* and $\frac{1}{2}$, by which it is named or called *half;* and the root of 3, by which it is called *the root of* 3; the like of any other.

3. Hence it is that unit is number. For the part is of the ſame matter that is its whole; the unit is part of the multitude of units, therefore the unit is of the ſame matter that is the multitude of units: but the matter of the multitude of units is number, therefore the matter of unit is number. For elſe, if from a number given no number be ſubtracted, the number given remaineth. Let three be the number given; from which number ſubtract or take away one, (which, as ſome conceive, is no number), therefore the number given remaineth; that is to ſay, there remaineth three: which is abſurd.

4. Hence it will be convenient to examine from whence number hath its riſe or beginning. Moſt authors maintain, that unit is the beginning of number, and itſelf no number: but looking upon the principles and definitions

in the firſt rudiments of geometry, we ſhall find that the definition of a point is no way congruous with the definition of an unit in arithmetic; and therefore one or unit muſt be in the bounds or limits of number, and conſequently the beginning of number is not to be found in the number one. Wherefore, when we make number and magnitude congruent in principles, and like in definitions, we make and conſtitute a cipher to be the beginning of number, or rather the medium between increaſing and decreaſing numbers, commonly called *abſolute* or *whole numbers*, and *negative* or *fractional numbers*. Than which nothing can be imagined more agreeable to the definition of a point in geometry. For as a point is an adjunct of a line, and itſelf no line; ſo is a (0) cipher an adjunct of number, and itſelf no number: and as a point in geometry cannot be divided or increaſed into parts; ſo likewiſe (0) cannot be divided or increaſed into parts. For as many points, though in number infinite, do make no line; ſo many (0) ciphers, though in number infinite, do make no number. For the line A B cannot be increaſed by the addition of the point C, neither can the number D be increaſed by the addition of the (0) cipher E; for if you add nothing to 6, the ſum will be 6, the (0) cipher neither increaſing nor diminiſhing the number 6: but if it be granted, that A B be extended or prolonged to the point C, ſo that A C be made a continued line; then A B is increaſed by the addition of the point C. In like manner, if we grant D (6) be prolonged to E (0), ſo that D E (60) be a continued number, making 60; then 6 is augmented by the aid of (0), as conſtituting the number (60) ſixty. And furthermore, that one or unit is material, and a number, and that (0) is the beginning of number, is proved by all authors, although indirectly: for the tables of ſines and tangents prove one degree to be a number; becauſe the ſine of 1 degree is 174524, (the radius being 10000000), and the beginning of that table is (0), and to it anſwereth 00000, &c.

A———B
C

D 6
E 0
—
Sum 6

A——B——C

D E } 60
6 0 }

5. Hence it is that nnmber is not quantity diſcontinued. For that which is but one quantity, is not quantity disjunct: (60) ſixty, as it is a number, is one quantity, *viz.* one number (60) ſixty; therefore, as it is number, it is not quantity disjunct. For number is ſome ſuch thing in magnitude, as humidity in water; for as humidity extends itſelf through all and every part of water, ſo number related to magnitude, doth extend itſelf through all and every part of magnitude. Alſo, as to continued water doth anſwer continued humidity, ſo to a continued magnitude doth anſwer a continued number: as the continued humidity of water ſuffereth the ſame diviſion and diſtinction that the water doth, ſo continued number ſuffereth the ſame diviſion and diſtinction that its magnitude doth. From all which conſiderations we might enlarge farther concerning number and magnitude, by comparing the definitions of the one with the principles of the other: for having found a (0) cipher to anſwer in definition to a point in magnitude, we may very well conclude, that number may be congruent to a line: as alſo a figurative number to be conſonant in definition with a ſuperficies and ſolid, *&c.* in the order of geometrical magnitudes.

6. The character or notes by which numbers are ſignified, or by which a number is ordinarily expreſſed, are theſe following, *viz.* 0, cipher, or nothing; 1, one; 2, two; 3, three; 4, four; 5, five, 6, ſix; 7, ſeven; 8, eight; 9, nine. The cipher, which though of itſelf it ſignifieth nothing, *viz.* expreſſeth not any certain or known quantity; yet it is the beginning, radix, or root of number; and the other nine characters are called *ſignificant figures*, or *digits*.

7. In numbers of any ſort, two things are to be conſidered, *viz.* notation and numeration.

8. Notation teacheth how to deſcribe any number by certain notes and characters, and to declare the value thereof being ſo deſcribed; and that is by degrees and periods.

9. A degree conſiſts of three figures, *viz.* of three places, comprehending units, tens, and hundreds. So 365 is a degree: and the firſt figure (5) on the right hand ſtands ſimply for its own value, being units, or ſo many

ones, *viz.* five; the ſecond in order from the right ſignifies as many times ten as there are units contained in it, *viz.* ſixty; the third in the ſame order ſignifies ſo many hundreds as it contains units; ſo will the expreſſion of the number be three hundred ſixty-five; alſo 789, is ſeven hundred eighty-nine, *&c.*

10. A period is, when a number conſiſts of more than three figures or places, according to whoſe proper order we are to prick or diſtinguiſh every third place, beginning at the right hand, and ſo on to the left. So the number 63452 being given, it will be diſtinguiſhed thus, 63,452, and expreſſed thus, ſixty-three thouſand, four hundred fifty-two. Likewiſe 4,578,236,782 being diſtinguiſhed as you ſee, will be expreſſed thus, four thouſand five hundred ſeventy-eight millions, two hundred thirty-ſix thouſand, ſeven hundred eighty-two.

11. Number is either abſolute or negative.

12. An abſolute, or entire, whole, increaſing number, is that which, by annexing another figure or cipher, becomes ten times ſo much as it ſtood for before; and if two figures or ciphers be annexed, it makes it an hundred times as much as it ſtood for before, *&c.*: As, if you annex to the figure 6 a cipher, then it will become (60) ſixty; ſo if two ciphers are annexed, then it will be (600) ſix hundred; and if you do annex to it a (4) four, then it will be (64) ſixty-four; and if you annex (78) ſeventy-eight, it will be then (678) ſix hundred ſeventy-eight; and ſo on. By annexing more figures or ciphers, it will increaſe in a decuple proportion *ad infinitum.*

13. A negative or broken, fractional, decreaſing number, is that which, by prefixing a point or prick towards the left hand, its value is decreaſed from ſo many units, to ſo many tenth parts of any thing; and if a point and (0) cipher or digit be prefixed, it will be then ſo many hundredth parts; and if a point and two ciphers or digits be prefixed, its value is decreaſed to be ſo many thouſandth parts: As if you would prefix before the figure 3 a point (.) or prick, thus (.3), it is then decreaſed from 3 units or 3 integers, to three tenth parts of an unit or integer; and if you prefix a point and cypher thus (.03), it is decreaſed from 3 integers to three hundred parts of an

integer: and by this means 5 *l.* abſolute, by prefixing of a point, will be decreaſed to 5 *l.* negative; which is 5 tenth parts of a pound, equal in value to ten ſhillings; and ſo by prefixing of more ciphers or digits, its value is decreaſed in a decuple proportion *ad infinitum:* As in the following ſcheme, or order of numbers, we have placed (0) cipher in its due place and order, as it is the beginning and medium of number; for going from (0) towards the left hand, you deal with entire, abſolute, whole, increaſing numbers.

Increaſing Numbers. *Decreaſing Numbers.*

876	543	256	2 1 0 1 2	345	678	976
			C.X.U.X.C.			
Thouſ. of millions.	Millions.	Thouſands.	Hundreds. Tens. Units. Tenth parts. Hund. parts.	Thouſ. parts.	Million parts.	Th. mill. parts.
C.X.	C.X.	C.X.		X.C.	X.C.	X.C.

But going from (0) the place of units, towards the right hand, you meet with broken, negative, fractional, and decreaſing numbers. And hence it follows, that multiplication increaſeth the product in abſolute numbers, but decreaſeth the product in negative numbers. Alſo diviſion decreaſeth the quotient in whole numbers, and increaſeth it in negative or fractional numbers.

14. An abſolute, entire, whole, increaſing number, hath always a point annexed towards the right hand; and therefore,

15. A negative, broken, decimal, decreaſing number, hath always a point prefixed before it towards the left hand. When we expreſs integers or whole numbers, as 5 pounds, 5 feet, 26 men, we uſually annex a
l. feet. men. inch.
point or prick after the number thus, 5. 5. 26. 347. But when we expreſs decimals, or decreaſing numbers, we do commonly prefix a point or prick before the ſaid decimal, or decreaſing number, thus (.3), that is three tenths, or 3 primes; (.03), that is, 3 hundredths, or 3 ſeconds.

16. A whole or abſolute number is an unit, or a compoſed multitude of units; and it is either a prime, or elſe a compoſite number.

17. Prime numbers amongſt themſelves, are thoſe which have no multitude of units for a common meaſurer, as 8 and 7, or 10 and 13; becauſe not any multitude of units can equally meaſure or divide them without a remainder.

18 Compoſite numbers amongſt themſelves, are thoſe which have a multitude of units for a common meaſurer, as 9 and 12: becauſe three meaſures them exactly, and abbreviates them to 3 and 4.

19. A broken number, commonly called a *fraction*, is a part or parts of a whole number, *viz.* a part of an integer; as ($\frac{1}{3}$), one third, is one third part of an unit.

20 A broken number or fraction conſiſts of two parts, *viz.* the numerator and denominator.

21. The numerator and denominator of a fraction are ſet one over the other, with a line between them; and the numerator is ſet above the line, and expreſſeth the number of parts therein contained.

22. The denominator of a fraction, is the inferior number placed below the line, and expreſſeth the number of parts into which the unit or integer is divided: As let $\frac{3}{4}$ be the fraction given, ſo ſhall 3 be the numerator, and doth expreſs or number the multitude of parts contained in this fraction; for $\frac{3}{4}$ is a fraction compounded of fourths or quarters, and the figure 3 in numbering, ſhews us, that in that fraction there are three of thoſe fourth parts or quarters: alſo in the ſame fraction $\frac{3}{4}$, 4 is the denominator, and doth expreſs the quality of the fraction, *viz.* that the whole or integer is divided into four equal parts.

23. A broken number is either proper or improper, *viz.* proper, when the numerator is leſs than the denominator; thus $\frac{3}{4}$ is a proper fraction: but an improper fraction hath its numerator greater, or at leaſt equal to the denominator; thus $\frac{9}{8}$ and $\frac{8}{8}$ are improper fractions.

24. A proper broken number is either ſimple or compound, *viz.* ſimple, when it hath one denominator, and compound, when it conſiſteth of divers denominators. If $\frac{3}{4}$, $\frac{9}{35}$, $\frac{35}{120}$, were given, we ſay they are each of them

ſingle or ſimple fractions, becauſe they conſiſt but of one numerator, and one denominator; but if $\frac{3}{4}$ of $\frac{9}{11}$ of $\frac{35}{110}$ of a pound Sterling were given, we ſay that it is a compound broken number or fraction, becauſe the expreſſion and repreſentation conſiſteth of more denominators than one. And ſuch by ſome are called *fractions of fractions*. They have always the particle *of* between them.

25. When a ſingle broken number or fraction hath for its denominator a number conſiſting of an unit in the firſt place toward the left hand, and nothing but ciphers from the unit toward the right hand, it is then aptly and rightly called a *decimal fraction*. Under this head are all our decreaſing numbers placed, and in our 13th definition called *negatives*. And by the order there preſcribed, we ſignify them to be decimals, by putting a point or prick before them, or before the numerator, rejecting the denominator. Therefore, according to our laſt rule, $\frac{5}{10}$, $\frac{5}{100}$, $\frac{25}{1000}$, are ſaid to be decimals: and a decimal fraction may be expreſſed without its denominator, as before by prefixing a point or prick before the numerator of the ſaid fraction, and then ſhall the former fractions $\frac{5}{10}$, $\frac{5}{100}$, and $\frac{25}{1000}$ ſtand thus, .5, .05, and .025.

But oftentimes, as in the ſecond and third fraction, $\frac{5}{100}$ and $\frac{25}{1000}$, a prick or point will not do without the help of a cipher or ciphers prefixed before the ſignificant figures of the numerator: and therefore when the numerator of a decimal fraction conſiſteth not of ſo many places as the denominator hath ciphers, fill up the void places of the numerator, by prefixing ciphers before the ſignificant figures of the numerator, and then ſign it for a decimal; ſo ſhall $\frac{5}{100}$ be .05, and $\frac{25}{1000}$ will be 025, and $\frac{72}{10000}$ will be .0072. Now, by this we may eaſily diſcover the denominator, having the numerator; for always the denominator of any decimal fraction conſiſts of ſo many ciphers as the numerator hath places, with an unit prefixed before the ſaid cipher, *viz.* under the point or prick.

26. A decimal number or fraction, is expreſſed by *primes*, *ſeconds*, *thirds*, *fourths*, &c.; and is number decreaſing. Here, inſtead of natural and common fractions, as $\frac{1}{4}$ of a thing, we diſtribute the thing or integer into

primes, *ſeconds*, *thirds*, *fourths*, *fifths*, &c. that our expreſſion may be conſonant to our former order.

27. In decimal arithmetic we always imagine, and it would be very commodious if it were really ſo, that all entire units, integers, and things, are divided, firſt, into ten equal parts; and theſe parts ſo divided we call *primes*: and, ſecondly, we divide alſo each of the former primes into other ten equal parts; and every one of theſe diviſions we call *ſeconds*: and, thirdly, we divide each of the ſaid ſeconds into ten other equal parts; and thoſe ſo divided we call *thirds*: and ſo by decimating the former, and ſubdecimating the latter, we run on *ad infinitum*.

28. Let a pound Sterling, Troy weight, Avoirdupois weight, liquid meaſure, dry meaſure, long meaſure, time, dozen, or any other thing, or integer, be given to be decimally divided; according to the notion premiſed, we ought to let the firſt diviſion be primes, the next diviſion ſeconds, the next thirds, *&c.* So one pound Sterling being 20 ſhillings, when divided into ten equal parts, the value of each part will be 2 ſhillings; therefore one prime of a pound Sterling will ſtand thus, (.1), which is in value 2 ſhillings; three primes will ſtand thus (.3), and that is in value 6 ſhillings. Again, a prime, or .1, being divided into ten equal parts, each of thoſe parts will be one ſecond, and is thus expreſſed, (.01); and its value will be found to be 2 d. farthing and $\frac{6}{10}$ of a farthing; and ſo will .05 ſignify one ſhilling, or five ſeconds. And if .01 be divided into ten other equal parts, each of thoſe parts ſo divided will be thirds, and will ſtand thus, .001; and its value will be found to be .96 of a farthing, or $\frac{96}{100}$ of a farthing; and .009 thirds will be 2 d. and .64 of a farthing, or $\frac{64}{100}$ of a farthing, *&c.* So that .375 l. will be found to repreſent 7 s. 6 d.; for the three primes are 6 ſhillings, and the ſeven ſeconds are 1 s. 4 d. and $\frac{8}{10}$ of a penny, and the five thirds are 1 penny and $\frac{2}{10}$ of a penny; which, added together, make 7 s. 6 d.

29. If you put any bulk or body repreſenting an integer, if it be decimally divided, then the parts in the firſt decimation are primes, the next ſeconds, and the next decimation is thirds, the next fourths, *&c.* As, let there

be given a bullet of lead, or ſuch like, whoſe weight let be 50 lb. Troy; this is called an *unit*, *integer*, or *thing*; then will the like weight and matter make 10 other, the which together, will be equal to 50 lb. and will weigh each of them 5 lb. apiece. Take of the ſame matter, equal to 5 lb. and make 10 more, then each of thoſe will weigh 6 ounces apiece. Alſo, if again you take 6 ounces, and thereof make 10 other ſmall bullets, each of them will weigh 12 penny weight Troy. And thus have you made primes, ſeconds, and thirds, in reſpect of the integer, containing 50 lb. Troy weight; ſo that five primes are equal to the half maſs, and two primes and five ſeconds is a quarter of the maſs; and therefore one of the firſt diviſion, 2 of the ſecond diviſion, and 5 of the third diviſion, will be equal in weight to half a quarter of the maſs, and will contain 6 lb. 3 oz.

30. When a decimal fraction followeth a whole number, you are to ſeparate or part the decimal from the whole number by a point or prick. So if .75 followed the whole number 32, ſet them thus, 32.75. You ſhall find that divers authors have divers ways in expreſſing mixed numbers; as thus, 32|75, or $32\frac{75}{100}$, or 3275; but you will find that 32.75, thus placed and expreſſed, is the fitteſt for calculation.

31. A mixed number hath two parts; the whole, and the broken. The whole is that which is compoſed of integers; and the broken is a fraction annexed thereunto. So the mixed number $36\frac{8}{12}$ being given, we ſay, that 36 is the whole number, which is compoſed of integers, and the $\frac{8}{12}$ is the broken number annexed; which ſheweth, that one of the former integers, (*viz.* one of 36), being divided into 12 parts, $\frac{8}{12}$ doth expreſs 8 of thoſe 12 parts more belonging to the ſaid 36 integers.

32. Denominative numbers are of one or of many. And thoſe are of divers ſorts and kinds, *viz.* ſingular, called *unit*, as 1; and plural, called *multitude*, as 2, 3, 4, 5: ſingle, of one kind only, called *digits*, as 1, 2, 3, 4, 5, 6, 7, 8, 9; and compounds of many, as 10, 11, 12, *&c.* 102, 367, *&c.*:

Proportional, as ſingle, multiple, double, triple, quadruple, *&c.*: denominate, as pounds, ſhillings, pence; undenominate,

denominate, as 1, 2, 3, *&c.* : perfect, as 6, 28, 496, 8128, 130816, 2096128, *&c.* whoſe parts are equal to the numbers; imperfect, unequal, and more than the ſum, as 12, to 1, 2, 3, 4, 6; imperfect, unequal, and leſs than the ſum, as 8, to 1, 2, 4: numbers commenſurable and incommenſurable, as 12 and 9 are commenſurable, becauſe 3 meaſures them both; but 6 and 17 are incommeaſurable, becauſe no one common number or meaſure can meaſure them: lineal, in form of a line, as ......; ſuperficial, in form of a ſuperficies or plane, as ::::::, or ⋮ ⋮, *&c.* and number cubical or ſolid in form of a cube. Theſe two latter are otherwiſe called *figurative numbers.* There are alſo other numbers called *tabular*, as ſines, tangents, ſecants, *&c.*; others that are called *logarithmetic*, or *borrowed numbers*, fitted to proportion for eaſe and ſpeedy calculation of all manner of queſtions.

CHAP. II.

Of the natural diviſion of integers, and the ſeveral denominations of the parts.

1. BEfore we come to calculation, or the ordering of numbers to operate any arithmetical queſtion propoſed, we will lay down tables of the denomination of ſeveral integers; and after that (having mentioned the ſeveral ſpecies and kinds of arithmetic) we ſhall immediately handle the ſpecies of numeration, which are the main pillars upon which the whole fabric of this art is built.

Of money, weights, &c.

2. The leaſt denomination or fraction of money uſed in England is a farthing, from whence is produced the following table, called *the table of coin*, viz.

1 Farth.	make	1 Farthing
4 Farth.		1 Penny
12 Pence		1 Shilling
20 Shill.		1 Pound

And therefore,

l.	*s.*	*d.*	*qrs.*
1	20	12	4
1	20	240	960
	1	12	48
		1	4

The firſt of theſe tables, *viz.* that on the left hand, is plain and eaſy to be underſtood, and therefore wants no directions. In the ſecond table, above the line you have 1 l. 20 s. 12 d. 4 qrs.; whereby is meant, that 1 pound is equal to 20 ſhillings, and 1 ſhilling is equal to 12 pence, and 1 penny is equal to 4 farthings. Under the line is 1 l. 20 s. 240 d. 960 qrs.; which ſignifies 1 pound to contain 20 ſhillings, or 240 pence, or 960 farthings. In the ſecond line below that is 1 s. 12 d. 48 qrs. the firſt ſtanding under the denomination of ſhillings; whereby is noted, that 1 ſhilling is equal to 12 pence, or 48 farthings. And likewiſe in the line below that, one penny is equal in value to four farthings. Underſtand the like reaſon in all the following tables of weight and meaſure, time, motion, and dozen.

Of Troy weight.

3. The leaſt fraction or denomination of weight uſed in England, is a grain of wheat gathered out of the middle of the ear, and well dried; from whence are produced theſe following tables of weight called *Troy weight.*

32 Grains of wheat	make	24 Artificial grains
24 Artificial grains	make	1 Penny-weight
20 Penny-weight	make	1 Ounce
12 Ounces	make	1 Pound Troy weight

And therefore,

lb.	*oz.*	*p. w.*	*grains.*
1	12	20	24
1	12	240	5760
	1	20	480
		1	24

Troy weight ſerveth to weigh bread, gold, ſilver, and electuaries. It alſo regulateth and preſcribeth a form how to keep the money of England at a certain ſtandard. The goldſmiths have divided the ounce Troy weight into other parts, which they generally call *mark-weight.* The denominative parts thereof are as followeth, *viz.* a mark (being an ounce Troy) is divided into 24 equal parts, called *carects*, and each carect into 4 grains; ſo that in a mark are 96 grains. By this weight they diſtinguiſh the different fineneſs of their gold; for if to 22 carects of gold be put 2 carects of alloy, (which is of ſilver, copper, or other baſer metal, with which they uſe to mix their gold or ſilver to abate the fineneſs thereof), both making when mixed but an ounce or 24 carects, then this gold is ſaid to be 22 carects fine; for if it come to be refined, the 2 carects of alloy will fly away, and leave only 22 carects of pure gold: the like to be conſidered of a greater or leſſer quantity. And as the fineneſs of gold is eſtimated by carects, ſo the fineneſs of ſilver is diſtinguiſhed by ounces: for if a pound of it be pure, and loſeth nothing in the refining, ſuch ſilver is ſaid to be twelve ounces fine; but if it loſeth any thing, it is ſaid to contain ſo much fineneſs as the loſs wanteth of twelve ounces; as if it loſt 1 ounce 14 penny-weight, then it is ſaid to be 10 ounces 6 penny-weight fine; and that which loſeth 2 ounces 4 penny-weight 16 grains, is ſaid to be 9 ounces 15 penny-weight 8 grains fine, &c.; the like of a greater or leſſer quantity.

Of apothecaries weight.

4. The apothecaries have their weights deduced from Troy weight, a pound Troy being the greateſt integer; a table of whoſe diviſion and ſubdiviſion followeth, *viz.*

1 Pound	makes	12 Ounces
1 Ounce		8 Drams
1 Dram		3 Scruples
1 Scruple		20 Grains

And therefore,

lb.	*oz.*	*dr.*	*ſcr.*	*gr.*
1	12	8	3	20
1	12	96	288	5760
	1	8	24	480
		1	3	60
			1	20

5. Thus much concerning Troy weight, and its derivative weights; which, as was ſaid before, ſerveth to weigh bread, gold, ſilver, and electuaries. Now, beſides Troy weight, there is another kind of weight uſed in England, commonly known by the name of *Avoirdupois weight*, (1 pound of which is equal to 14 ounces 12 penny-weight Troy weight); and it ſerveth to weigh

all kind of grocery wares, as alſo butter, cheeſe, fleſh, wax, tallow, roſin, pitch, lead, and all ſuch kind of garble; the table of which weight is as followeth.

The table of Avoirdupois weight.

4 Quarters of a dram	make	1 Dram
16 Drams		1 Ounce
16 Ounces		1 Pound
28 Pounds		1 Quarter of a hundred
4 Quarters		1 Hund^{d} weight or 112 lb.
20 Hundred		1 Tun

And therefore,

Tun.	*C.*	*qrs.*	*lb.*	*oz.*	*dr.*	*qrs.*
1	20	4	28	16	16	4
1	20	80	2240	35840	573440	2293760
	1	4	112	1792	28672	114688
		1	28	448	7168	28672
			1	16	256	1024
				1	16	64
					1	4

Wool is weighed with this weight, but only the diviſions are not the ſame; a table whereof followeth.

A table of the denominative parts of wool-weight.

7 Pound	make	1 Clove
2 Cloves		1 Stone
2 Stones		1 Todd
6 Todd 1 ſtone		1 Wey
2 Weys		1 Sack
12 Sacks		1 Laſt

And therefore,

Laſt.	*ſack.*	*wey.*	*todd.*	*ſtone.*	*cloves.*	*lb.*
1	12	2	6½	2	2	7
1	12	24	156	312	624	44368
	1	2	13	26	52	364
		1	6½	13	26	182
			1	2	4	28
				1	2	14
					1	7

Note, That in ſome counties the wey is 256 lb. Avoirdupois, as in the Suffolk wey; but in Eſſex there is 336 lb. in a wey.

6. The leaſt denominative part of liquid meaſure is a pint, which was formerly taken from Troy weight, (1 lb. of wheat Troy weight making a pint of liquid meaſure): but in regard of the difference between the brewers and farmers of his Majeſty's exciſe, concerning the gauging of veſſels, occaſioned by the different opinions of artiſts concerning the ſolid inches in a gallon, it was lately decided by act of parliament; the ſtatute now making 282 ſolid inches in a beer gallon, and 231 in a wine gallon; and conſequently the pint beer meaſure to contain $35\frac{1}{4}$ ſolid inches, and the pint wine meaſure to contain $28\frac{7}{8}$ cubical or ſolid inches. From whence is drawn the following table.

The table of liquid meaſure.

$35\frac{1}{4}$ Cubical inches	make	1 Pint beer meaſure
$28\frac{7}{8}$ Cubical inches		1 Pint wine meaſure
2 Pints		1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
8 Gallons		1 Firkin of ale, ſoap, or herring
9 Gallons		1 Firkin of beer
10 Gallons and a half		1 Firk. of ſalmon or eels
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrel
42 Gallons		1 Tierce of wine
63 Gallons		1 Hogſhead
2 Hogſheads		1 Pipe or butt
2 Pipes or butts		1 Tun of wine.

And therefore,

Tuns.	*pipes.*	*hhds.*	*gall.*	*pints.*
1	2	2	63	8
1	2	4	252	2016
	1	2	126	1008
		1	63	504
			1	8

7. The leaſt denominative part of dry meaſure is alſo a pint, and this is likewiſe taken from Troy weight: the table of whoſe diviſion followeth,

The table of dry meaſure.

1 Pound Troy	make	1 Pint
2 Pints		1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Buſhel
4 Buſhels		1 Comb
2 Combs		1 Quarter
4 Quarters		1 Chaldron
5 Quarters		1 Wey
2 Weys		1 Laſt

And therefore,

Laſt.	*wey.*	*qrs.*	*com.*	*buſh*	*pecks.*	*gall.*	*pints.*
1	2	5	2	4	4	2	8
1	2	10	20	80	320	640	5120
	1	5	10	40	160	320	2560
		1	2	8	32	64	512
			1	4	16	32	256
				1	4	8	64
					1	2	16
						1	8

8. The leaſt denominative part of long meaſure is a barley-corn well dried, and taken out of the middle of the ear; whoſe table of parts followeth.

3 Barley-corns	make	1 Inch
12 Inches		1 Foot
3 Feet		1 Yard
3 Feet 9 inches, or a yard and a quarter		1 Ell Engliſh
6 Feet		1 Fathom
5 Yards and a half		1 Pole, perch, or rod
40 Poles or perches		1 Furlong
8 Furlongs		1 Engliſh mile

And therefore,

Mile	*furl.*	*poles.*	*yards.*	*feet.*	*inches.*	*barly-corns.*
1	8	40	5½	3	12	3
1	8	320	1760	5280	63360	190080
	1	40	220	660	7920	23760
		1	5½	16½	198	594
			1	3	36	108
				1	12	36
					1	3

And note, That the yard, as alſo the ell, is uſually divided into 4 quarters, and each quarter into 4 nails.

Note alſo, That a geometrical pace is 5 feet; and there are 1056 ſuch paces in an Engliſh mile.

9. The parts of the ſuperficial meaſures of land, are ſuch as are mentioned in the following table, *viz.*

A table of land meaſure.

40 Square poles or perches	make	1 Rood or quarter of an acre
4 Roods		1 Acre

By the foregoing table of long meaſure, you are informed what a pole, or, which is all one, a perch, is; and by this, that 40 ſquare perches is a rood. Now, a ſquare perch is a ſuperficies very aptly reſembled by a ſquare trencher, every ſide thereof being a perch of 5½ yards in length, 40 of them is a rood, and 4 roods an acre: ſo that a ſuperficies that is 40 perches long and 4 broad, is an acre of land, the acre containing in all 160 ſquare perches.

10. The leaſt denominative part of time is 1 minute, the greateſt integer being a year; from whence is produced this following table.

The table of time.

1 Minute	make	1 Minute
60 Minutes		1 Hour
24 Hours		1 Day natural
7 Days		1 Week
4 Weeks		1 Month
13 Months, 1 day, 6 hours		1 Year

But the year is uſually divided into 12 unequal calendar months; whoſe names, and the number of days they contain, follow, *viz.*

	Days.
January	31
February	28
March	31
April	30
May	31
June	30
July	31
Auguſt	31
September	30
October	31
November	30
December	31
	365

So that the year containeth 365 days and 6 hours; but the 6 hours are not reckoned, but only every 4th year, and then there is a day added to the latter end of February, and then it containeth 29 days, and that year is called *leap year*, and containeth 366 days.

And here note, That as the hour is divided into 60 minutes, ſo each minute is ſubdivided into 60 ſeconds, and each ſecond into 60 thirds, and each third into 60 fourths, *&c.*

The tropical year, by the exacteſt obſervation of the moſt accurate aſtronomers, is found to be 365 days, 5 hours, 49 minutes, 4 ſeconds, and 21 thirds.

CHAP. III.

Of the ſpecies or kinds of arithmetic.

1. ARithmetic is either natural, artificial, analytical, algebraical, lineal, or inſtrumental.

2. Natural arithmetic is that which is performed by the numbers themſelves. And this is either poſitive or negative. Poſitive, which is wrought by certain infallible numbers propounded. And this is either ſingle or comparative. Single, which conſidereth the nature of numbers ſimply by themſelves; and comparative, which is wrought by numbers as they have relation to one another.

ther. And the negative part relates to the rule of falſe.

3. Artificial (by ſome called logarithmetical) arithmetic, is that which is performed by artificial or borrowed numbers invented for that purpoſe, which are called *logarithms*.

4. Analytical arithmetic is that which ſhews how from a thing unknown to find truly that which is ſought, always keeping the ſpecies without change.

5. Algebraical arithmetic is an obſcure and hidden art of accounting by numbers, in reſolving of hard queſtions.

6. Lineal arithmetic is that which is performed by lines fitted to proportion, and geometrical projections.

7. Inſtrumental arithmetic is that which is performed by inſtruments fitted with circular and right lines of proportion, by the motion of an index, or otherwiſe.

8. The parts of ſingle arithmetic are, numeration, and the extraction of roots.

9. Numeration is that which, by certain known numbers propounded, diſcovers another number unknown.

10. Numeration hath four ſpecies, *viz.* addition, ſubtraction, multiplication, and diviſion.

CHAP. IV.

Addition of whole numbers.

1. ADdition is the reduction of two or more numbers of like kind together into one ſum or total; or it is that by which divers numbers are added together, to the end that the ſum or total value of them all may be diſcovered.

The firſt number in every addition is called *the addible number;* the other, *the number* or *numbers added;* and the number invented by the addition, is called the *aggregate* or *ſum,* containing the value of the addition.

The collation of the numbers, is the right placing the numbers given reſpectively to each denomination; and the operation is the artificial adding of the numbers given together, in order to the finding out of the aggregate or ſum.

2. In addition, place the numbers given reſpectively the one above the other, in ſuch ſort, that the like degree, place, or denomination, may ſtand in the ſame ſeries, *viz.* units under units, tens under tens, hundreds under hundreds, *&c.*; pounds under pounds, ſhillings under ſhillings, pence under pence, *&c.*; yards under yards, feet under feet, *&c.*

3. Having thus placed the numbers given, (as before directed), and drawn a line under them, add them together, beginning with the leſſer denomination, *viz.* at the right hand, and ſo on, ſubſcribing the ſum under the line reſpectively. As for example:

Let there be given 3352, and 213, and 133, to be added together. I ſet the units in each particular number under each other, and ſo likewiſe the tens under the tens, *&c.* and draw a line under them, as in the margin. Then I begin at the place of units, and add them together upwards; ſaying, 3 and 3 are 6, and 2 makes 8; which I ſet under the line, and under the figures added together. Then I proceed to the next place, being the place of tens, and add them up in the ſame manner as I did the place of units; ſaying, 3 and 1 are four, and 5 are 9; which I likewiſe ſet under the line reſpectively. Then I go to the place of hundreds, and add them up as I did the other; ſaying, 1 and 2 are 3, and 3 are 6; which I alſo ſet under the line. And, laſtly, I go to the place of thouſands, and, becauſe there are no other figures to add to the 3, I ſet it under the line in its reſpective place: and ſo the work is finiſhed; and I find the ſum of the three given numbers to be 3698.

$$\begin{array}{r}3352\\213\\133\\\hline 3698\end{array}$$

4. But if the ſum of the figures of any ſeries exceedeth ten, or any number of tens, ſubſcribe under the ſame the exceſs above the tens; and for every ten carry one to be added to the next ſeries towards the left hand, and ſo go on, till you have finiſhed your addition; always remembering, that how great ſoever the ſum of the figures of the laſt ſeries is, it muſt all be ſet down under the line reſpectively. So 3678 being given to be added

to 2357, I ſet them down as is before directed, and as you ſee in the margin, with a line drawn under them. Then I begin and add them together; ſaying, 7 and 8 are 15; which is 5 above 10: wherefore I ſet 5 under the line, and carry 1 for the 10, to be added to the next ſeries; ſaying, 1 that I carried and 5 is 6, and 7 are 13; wherefore I ſet down 3, and carry 1 for the 10 to the next ſeries. Then I ſay, 1 that I carried and 3 are 4, and 6 are 10. Now, becauſe it comes to juſt 10, and no more, I ſet 0 under the line, and carry 1 for the 10 to the next, and ſay, 1 that I carried and 2 are 3, and 3 are 6; which I ſet down in its reſpective place. Thus the addition is ended, and the total ſum of theſe numbers is found to be 6035. Several examples of this kind follow.

$$\begin{array}{r}3678\\2357\\\hline 6035\end{array}$$

$$\text{Numbers to be added}\left\{\begin{array}{r}354867\\573846\\785946\\347205\end{array}\right.$$

$$\text{Sum - - }\quad 2061864$$

$$\text{Numbers to be added}\left\{\begin{array}{r}748647\\465834\\76483\\648300\end{array}\right.$$

$$\text{Sum - - }\quad 1939264$$

$$\text{Numbers to be added}\left\{\begin{array}{r}45346\\38074\\8437\\923\\76\end{array}\right.$$

$$\text{Sum - - }\quad 92856$$

5. If the numbers given to be added are contained under divers denominations, as of *pounds, ſhillings, pence,* and *farthings*; or of *tuns, hundreds, quarters, pounds,* &c.; then, in this caſe, having diſpoſed of the numbers, each denomination under other of the like kind; beginning at the leaſt denomination, (minding how many of one denomination do make an integer in the next), and having added them up; for every integer of the next greater denomination that you find therein contained, bear an unit in mind to be added to the ſaid next greater denomination, expreſſing the exceſs reſpectively under the line. Proceed in this manner until your addition be finiſhed. The following example will make the rule plain to the learner,

learner. Thus theſe following ſums being given to be added, *viz.* 136 l. 13 s. 4 d. 2 qrs. and 79 l. 7 s. 10 d. 3 qrs. and 33 l. 18 s. 9 d. 1 qr. alſo 15 l. 9 s. 5 d. 0 qrs.; the numbers being diſpoſed according to order, will ſtand as in the margin. Then I begin at the denomination of farthings, and add them up; ſaying, 1 and 3 are 4, and 2 makes 6. Now I conſider that 6 farthings are 1 penny and 2 farthings; wherefore I ſet down the 2 farthings in its place under the line, and keep 1 in mind to be added to the next denomination of pence. Then I go on, ſaying, 1 that I carried and 5 are 6, and 9 are 15, and 10 are 25, and 4 are 29. Now, I conſider that 29 pence are 2 ſhillings and 5 pence; therefore I ſet the 5 pence in order under the line, and keep 2 in mind for the 2 ſhillings, to be added to the ſhillings. Then I go on, ſaying, 2 that I carried and 9 are 11, and 18 are 29, and 7 are 36, and 13 are 49. Then I conſider that 49 ſhillings are 2 pounds and 9 ſhillings; wherefore I ſet the 9 ſhillings under the line, and carry the 2 for the 2 pounds to the next and laſt denomination of pounds; and proceed, ſaying, 2 that I carried and 5 make 7, and 3 are 10, and 9 are 19, and 6 are 25. Then I ſet down 5, and carry 2 for the two tens, and proceed; ſaying, 2 that I carried and 1 is 3, and 3 are 6, and 7 are 13, and 3 make 16. And I ſet down 6, and carry 1 for the 16, and go on; ſaying, 1 that I carried and 1 are 2; which I ſet in its place under the line, and the work is finiſhed. And thus I find the ſum of the foreſaid numbers to be 265 l. 9 s. 5 d. 2 qrs. This to the ingenious practitioner is ſufficient. But I ſhall, for the further illuminating of the weaker apprehenſions, explain the operation of another example in Troy weight. And here the learner muſt take notice of the table of Troy weight mentioned or ſet down in the third ſection of the ſecond chapter. The numbers given in this example are, 38 l. 7 oz. 13 p. w. 18 gr. and 50 lb. 10 oz. 10 p. w. 12 gr. and 42 lb. 8 oz. 5 p. w. 16 gr. And, in order to the addition thereof, I place them as you ſee, and proceed to operation; ſay-

l.	*s.*	*d.*	*qrs.*
136	13	4	2
79	7	10	3
33	18	9	1
15	9	5	0
265	9	5	2

ing, 16 and 12 are 28, and 18 are 46. Now, becauſe 24 grains make 1 penny-weight, 46 grains are 1 penny-weight and 22 grains; wherefore I ſet down 22, and carry 1 for the penny-weight; and, going on, I ſay, 1 that I carry and 5 makes 6, and 10 are 16, and 13 are 29; which is 1 ounce and 9 penny-weight. I ſet down 9 in its place under the line, and carry 1 to the ounces; ſaying, 1 that I carry and 8 are 9, and 10 are 19, and 7 are 26. And becauſe 26 ounces make 2 pounds 2 ounces, I ſet down 2 for the ounces, and carry 2 to the pounds: going on, 2 that I carry and 2 are 4, and 8 makes 12; that is 2 and go 1: then 1 I carry and 4 are 5, and 5 are 10, and 3 are 13; which I ſet down as in the margin, and the work is finiſhed: and I find the ſum of the ſaid numbers to amount to 132 lb. 2 oz. 9 p. w. 22 gr. This is ſufficient for the underſtanding of the following examples, or any other that ſhall come to thy view. The way of proving theſe, or any ſum in this rule, is ſhewed immediately after the enſuing examples.

lb.	*oz.*	*pw.*	*gr.*
38—	7—	13—	18
50—	10—	10—	12
42—	8—	5—	16
132—	2—	9—	22

Addition of Engliſh money.

l.	*s.*	*d.*	*qrs.*
436—	13—	7—	1
184—	9—	10—	3
768—	17—	4—	2
564—	11—	11—	0
1954—	12—	9—	2

l.	*s.*	*d.*	*qrs.*
48—	15—	11—	1
76—	10—	7—	3
18—	0—	5—	3
24—	19—	9—	2
168—	6—	10—	1

Addition of Troy weight.

lb.	*oz.*	*pw.*	*gr.*
15—	7—	13—	12
18—	6—	4—	20
11—	10—	16—	18
9—	4—	10—	22
19—	11—	18—	4
22—	0—	0—	0
97—	5—	4—	4

lb.	*oz.*	*pw.*	*gr.*
145—	9—	12—	18
726—	8—	14—	10
389—	7—	6—	13
83—	10—	16—	20
130—	0—	10—	12
74—	7—	15—	0
1550—	8—	10—	11

Addition of apothecaries weight.

lb.	*oz.*	*dr.*	*ſc.*	*gr.*
48	7	1	0	14
74	5	5	2	10
64	10	7	1	16
17	8	1	0	11
34	9	6	1	9
240	5	6	1	0

lb.	*oz.*	*dr.*	*ſc.*	*gr.*
60	3	4	0	10
48	10	6	0	14
34	8	2	1	15
18	11	2	2	11
160	7	1	2	15
35	2	5	1	7
358	7	7	0	12

Addition of Avoirdupois weight.

Tuns.	*C.*	*qrs.*	*lb.*
75	13	1	15
48	7	3	21
60	11	1	17
21	7	0	25
12	16	0	11
218	17	0	5

lb.	*oz.*	*dr.*
36	10	12
22	11	13
11	7	4
15	4	10
20	10	9
106	13	0

Addition of liquid meaſure.

Tuns.	*pipes.*	*hhds.*	*gall.*
45	1	1	48
15	0	1	17
38	0	0	47
12	1	0	56
21	1	1	18
133	0	1	60

Tuns.	*hhds.*	*gall.*	*pints.*
30	3	40	4
12	0	28	6
47	5	60	5
57	3	22	3
17	0	0	0
168	1	26	2

Addition of dry meaſure.

Chald.	*qrs.*	*buſh.*	*pec.*
48	3	7	3
13	1	4	0
54	0	6	2
16	3	6	1
40	1	0	1
173	3	0	3

qrs.	*buſh.*	*pec.*	*gall.*
17	3	1	1
50	1	3	0
14	5	3	1
40	2	0	1
30	0	3	0
152	5	3	1

Addition of long meaſure.

Yards.	*qrs.*	*nails.*	*Ells.*	*qrs.*	*nails.*
35	3	3	56	1	3
14	1	2	13	3	2
74	2	3	48	2	1
38	0	1	50	1	0
30	1	0	74	0	2
15	0	0	17	1	0
208	1	1	260	0	0

Addition of land meaſure.

Acres.	*roods.*	*perch.*	*Acres.*	*roods.*	*perch.*
12	3	18	86	1	36
14	0	24	47	3	24
30	2	19	73	2	18
48	3	30	60	0	7
28	1	38	4	2	8
50	3	26	14	1	14
185	3	35	286	3	27

The proof of addition.

6. Addition is proved after this manner. When you have found out the ſum of the number given, then ſeparate the uppermoſt line from the reſt with a ſtroke or daſh of the pen, and then add them all up again as you did before, leaving out the uppermoſt line; and having ſo done, add the new-invented ſum to the uppermoſt line you ſeparated; and if the ſum of thoſe two lines be equal to the ſum firſt found out, then the work was performed true, otherwiſe not. As for example: Let us prove the firſt example of addition of money, whoſe ſum we found to be 265 l. 9 s. 5 d. 2 qrs.; and

which we prove thus. Having ſeparated the uppermoſt number from the reſt by a line, as you ſee in the margin; then I add the ſame together again, leaving out the ſaid uppermoſt line, and the ſum thereof I ſet under the firſt ſum or true ſum, which doth amount to 128 l. 16 s. 1 d. 0 qrs. then again I add this new ſum to the uppermoſt line that before was ſeparated from the reſt, and the ſum of thoſe two is 265 l. 9 s. 5 d. 2 qrs. the ſame with the firſt ſum; and therefore I conclude that the operation was rightly performed.

l.	*s.*	*d.*	*qrs.*
136	13	04	2
79	07	10	3
33	18	09	1
15	09	05	0
265	09	05	2
128	16	01	0
265	09	05	2

7. The main end of addition in queſtions reſolvable thereby, is to know the ſum of ſeveral debts, parcels, integers, *&c.* Some queſtions may be theſe that follow.

Queſt. 1. There was an old man whoſe age was required. To which he replied, I have ſeven ſons, each having two years between the birth of each other; and in the 44th year of my age my eldeſt ſon was born, which is now the age of the youngeſt. I demand, What was the old man's age?

Now, to reſolve this queſtion, firſt ſet down the father's age at the birth of his firſt child, which was 44; then the difference between the oldeſt and the youngeſt, which is 12 years; and then the age of the youngeſt, which is 44: and then add them all together; and their ſum is 100, the complete age of their father.

44
12
44
100

Queſt. 2. A man lent his friend at ſeveral times theſe ſeveral ſums, *viz.* at one time 63 l. at another time 50 l. at another time 48 l. at another time 156 l. Now I deſire to know how much was lent him in all?

Set the ſums lent one under another, as you ſee in the margin; and then add them together, and you will find their ſum to amount to 317 l. which is the total of all the ſeveral ſums lent, and ſo much is due to the creditor.

63
50
48
156
317

Queſt. 3. From London to Ware is 20 miles, thence to Huntington 29 miles, thence to Stamford 21 miles, thence to Tuxford 36 miles, thence to Wentbridge 25 miles, from thence to York 20 miles. Now I deſire to know how many miles it is from London to York, according to this reckoning?

Now, to anſwer this queſtion, ſet down the ſeveral diſtances given, as you ſee in the margin; and add them together, and you will find their ſum to amount to 151; which is the true diſtance in miles between London and York.

20
29
21
36
25
20
——
151

Queſt. 4. There are two numbers, the leaſt whereof is 40, and their difference 14. I deſire to know what is the greater number, and alſo what is the ſum of them both? Firſt ſet down the leaſt, *viz.* 40, and 14, the difference; and add them together, and their ſum is 54 for the greateſt number. Then I ſet 40 (the leaſt) under 54 (the greateſt) and add them together, and their ſum is 94, equal to the greateſt and leaſt numbers.

40
14
—
Greateſt 54
Leaſt 40
—
Sum - 94

C H A P. V.

Of ſubtraction of whole numbers.

1. SUbtraction is the taking of a leſſer number out of a greater of a like kind, whereby to find out a third number, being or declaring the inequality, exceſs or difference between the numbers given. Or, ſubtraction is that by which one number is taken out of another number given, to the end that the reſidue or remainder may be known; which remainder is alſo called the *reſt*, *remainder*, or *difference* of the numbers given.

2. The number out of which ſubtraction is to be made, muſt be greater, or at leaſt equal with the other number given. The higher or ſuperior number is called

the *major* number; and the lower or inferior is called the *minor* number; and the operation of ſubtraction being finiſhed, the reſt or remainder is called the *difference* of the numbers given.

3. In ſubtraction, place the numbers given reſpectively the one under the other, in ſuch ſort as like degrees, places, or denominations, may ſtand in the ſame ſeries, *viz.* units under units, tens under tens, pounds under pounds, *&c.* feet under feet, and parts under parts, *&c.* This being done, draw a line underneath, as in addition.

4. Having placed the numbers given as is before directed, and drawn a line under them, ſubtract the lower number (which in this caſe muſt always be leſs than the uppermoſt) out of the higher number, and ſubſcribe the difference or remainder reſpectively below the line; and when the work is finiſhed, the number below the line will give you the remainder.

As for example: Let 364521 be given to be ſubtracted from 795836. I ſet the leſſer under the greater, as in the margin, and draw a line under them; then beginning at the right hand, I ſay, 1 out of 6, and there remains 5, which I ſet in order under the line. Then I proceed to the next, ſaying, 2 from 3 reſts 1; which I note alſo under the line. And thus I go on till I have finiſhed the work. And then I find the remainder or difference to be 431315.

$$\begin{array}{r}795836\\364521\\\hline 431315\end{array}$$

5. But if it ſo happen, as commonly it doth, that the lowermoſt number or figure is greater than the uppermoſt; then, in this caſe, add 10 to the uppermoſt number, and ſubtract the ſaid lowermoſt number from their ſum, and the remainder place under the line; and when you go to the next figure below, pay an unit, by adding it thereto for the 10 you borrowed before, and ſubtract that from the higher number of figures. And thus go on till your ſubtraction be finiſhed. As for example: Let 437503 be given, from whence it is required to ſubtract 153827, I diſpoſe of the numbers as is before directed, and as you ſee in the margin; then I begin, ſaying, 7 from 3 I cannot, but, adding 10 thereto, I ſay, 7 from 13, and there remains 6; which I ſet under the line in order. Then

I proceed to the next figure, ſaying, 1 that I borrowed and 2 is 3 from 0 I cannot, but 3 from 10, and there remains 7; which I likewiſe ſet down as before. Then 1 that I borrowed and 8 is 9 from 5 I cannot, but 9 from 15, and there remains 6. Then 1 I borrowed and 3 is 4 from 7, and there remains 3. Then 5 from 3 I cannot, but 5 from 13, and there remains 8. Then 1 I borrowed and 1 are 2 from 4, and there reſts 2. And thus the work is finiſhed. And after theſe numbers are ſubtracted one from another, the inequality, remainder, exceſs, or difference, is found to be 283676. Examples for thy farther experience may be theſe that follow.

437503
153827
———
283676

From	3469916	From	361576
Take	738642	Take	5864
Reſts	2731274	Reſts	355712

6. If the ſum or number to be ſubtracted is of ſeveral denominations, place the leſſer ſum below the greater, and in the ſame rank and order as is ſhewed in addition of the ſame numbers. Then begin at the right hand, and take the lower number out of the uppermoſt, if it be leſſer; but if it be bigger than the uppermoſt, then borrow an unit from the next greater denomination, and turn it into the parts of the leſs denomination, and add thoſe parts to the uppermoſt, noting the remainder below the line. Then proceed, and pay 1 to the next denomination for that which you borrowed before; and proceed in this order, until the work be finiſhed. An example of this rule may be this that followeth. Let 375 l. 13 s. 7 d. 1 qr. be given, from whence let it be required to ſubtract 57 l. 16 s. 3 d. 2 qrs. In order whereunto I place the numbers as you ſee in the margin. And thus I begin at the leaſt denomination, ſaying, 2 from 1 I cannot, therefore I borrow 1 penny from the next denomination, and turn it into farthings, which is 4; and adding 4 to 1, which is 5, I ſay,

l.	*s.*	*d.*	*q.*
375—	13—	7—	1
57—	16—	3—	2
317—	17—	3—	3

2 from 5, and there remains 3; which I put under the line. Then going on, I ſay, 1 that I borrowed and 3 is 4 from 7, and there reſts 3. Then going on, I ſay, 16 from 13 I cannot, but borrowing 1 pound, and turning it into 20 ſhillings, I add it to 13, and that is 33; wherefore I ſay, 16 from 33, and there remains 17; which I ſet under the line; and go on, ſaying, 1 that I borrowed and 7 is 8 from 5 I cannot, but 8 from 15, and there remains 7; the 1 that I borrowed and 5 is 6 from 7, there reſts 1, and 0 from 3 reſts 3. And the work is done. And I find the remainder or difference to be 317 l. 17 s. 3 d. 3 qrs.

An example of Troy weight may be this. I would ſubtract 17 lb. 10 oz. 11 pw. 20 gr. from 24 lb. 5 oz. 0 pw. 8 gr. I place the numbers according to the rule; and begin, ſaying 20 from 8 I cannot, but borrow 1 penny-weight, which is 24 grains, and add them to 8, and they are 32; wherefore I ſay, 20 from 32 reſts 12. Then 1 that I borrowed and 11 is 12 from 0 I cannot, but 12 from 20, borrowing an ounce, which is 20 penny-weight, and there remains 8. Then 1 that I borrowed and 10 is 11 from 5 I cannot, but 11 from 17, and there reſts 6. Then 1 that I borrowed and 7 is 8 from 4 I cannot, but 8 from 14, and there reſts 6. Then 1 that I borrowed and 1 is 2 from 2, and there reſts nothing. So that I find the remainder or difference to be 6 lb. 6 oz. 8 pw. 12 gr.

lb.	*oz.*	*pw.*	*gr.*
24	5	0	8
17	10	11	20
6	6	8	12

7. It many times happeneth, that you have many ſums or numbers to be ſubtracted from one number, as ſuppoſe a man ſhould lend his friend a certain ſum of money, and his friend hath paid him part of his debt at ſeveral times. Then, before you can conveniently know what is ſtill owing, you are to add the ſeveral numbers or ſums of payments together, and ſubtract their ſum from the whole debt, and the remainder is the ſum due to the creditor. As ſuppoſe A lendeth to B 564 l. 16 s. 10 d. and B hath repaid him 79 l. 16 s. 8 d. at one time, and 163 l. 18 s. 11 d. at another

other time, and 241 l. 15 s. 8 d. at another time; and you would know how the account ſtandeth between them, or what more is due to A. In order whereunto I firſt ſet down the ſum which A lent, and draw a line underneath it; then under that line I ſet the ſeveral ſums of payment as you ſee in the margin; and having brought the ſeveral ſums of payment into one total, by the fifth rule of the fourth chapter foregoing, I find their ſum amounteth to 485 l. 11 s. 3 d. which I ſubtract from the ſum firſt lent by A, by the ſixth rule of this chapter; and I find the remainder to be 79 l. 5 s. 7 d. and ſo much is ſtill due to A.

	l.	*s.*	*d.*
Lent -	564	16	10
Paid at ſeveral payments	79	16	8
	163	18	11
	241	15	8
Paid in all	485	11	3
Remains -	79	5	7

When the learner hath good knowledge of what hath been already delivered in this and the foregoing chapters, he will with eaſe underſtand the manner of working the following examples.

Subtraction of money.

	l.	*s.*	*d.*	*l.*	*s.*	*d.*	*q.*
Borrowed - -	374	10	3	700	10	11	2
Paid - - -	79	15	11	9	3	11	3
Remains - -	294	14	4	691	6	11	3

	l.	*s.*	*d.*	*l.*	*s.*	*d.*	*q.*
Borrowed - -	1000	0	0	711	3	0	0
Paid - - -	19	0	6	11	13	0	1
Rem. due -	980	19	6	699	9	11	3

Borrowed

	l.	*s.*	*d.*	*qrs.*
Borrowed - - - -	3300	0	0	0
Paid at ſeveral payments	170	10	0	0
	361	13	10	1
	590	3	4	3
	73	4	11	3
Paid in all - -	1195	12	2	3
Remains due - -	2104	7	9	1

Subtraction of Troy weight.

	l.	*oz.*	*pw.*	*gr.*
Bought - - - -	174	0	13	0
Sold - - - - -	78	4	16	15
Remains - - - -	95	7	16	9

	lb.	*oz.*	*pw.*	*gr.*
Bought - - -	470	10	13	0
Sold at ſeveral times	60	0	0	0
	35	10	18	0
	16	7	9	8
	48	4	0	0
	61	11	19	23
	23	0	0	0
Sold in all - -	245	10	7	7
Remains unſold -	225	0	5	17

Subtraction of apothecaries weight.

	lb.	*oz.*	*dr.*	*ſc*	*gr.*	*lb.*	*oz.*	*dr.*	*ſc.*	*gr.*
Bought	12	4	3	0	0	20	0	1	0	7
Sold -	8	5	1	1	15	10	0	1	2	12
Remains	3	11	1	1	5	9	11	7	0	15

Subtraction of Avoirdupois weight.

	C.	*qrs.*	*lb.*	*Tu.*	*C.*	*qrs.*	*lb.*	*oz.*	*dr.*
Bought -	35	0	15	5	7	1	10	10	5
Sold - -	16	2	20	3	17	1	16	9	13
Remains -	18	1	23	1	9	3	22	0	8

Subtraction of liquid meaſure.

	Tuns.	*hhd.*	*gall.*	*Tuns.*	*hhd.*	*gall.*	*pints.*
Bought	40	1	30	60	3	42	4
Sold	16	1	40	15	3	46	6
Remains	23	3	53	44	3	58	6

Subtraction of dry meaſure.

	Chal.	*qrs.*	*buſh.*	*pec.*	*Chald.*	*qrs.*	*buſh.*	*pec.*
Bought	100	0	0	0	73	2	3	2
Sold	54	1	4	3	46	2	3	3
Remains	45	2	3	1	26	3	7	3

Subtraction of long meaſure.

	Yards.	*qrs.*	*nails.*	*Yards.*	*qrs.*	*nails.*
Bought	160	0	0	344	0	1
Sold -	64	1	2	177	1	3
Remains	95	2	2	166	2	2

Subtraction of land meaſure.

	Acres.	*rood.*	*perch.*	*Acres.*	*roods.*	*perch.*
Bought	140	2	13	600	0	0
Sold	70	3	12	54	0	16
Remains	69	3	1	545	3	34

The

The proof of ſubtraction.

8. When your ſubtraction is ended, if you deſire to prove the work, whether it be true or no, then add the remainder to the minor number; and if the aggregate of theſe two be equal to the major number, then is your operation true; otherwiſe falſe. Thus let us prove the firſt example of the fifth rule of this chapter, where, after ſubtraction is ended, the numbers ſtand as in the margin; the remainder or difference being 283676. Now, to prove the work, I add the ſaid remainder 283676 to the minor number 153827, by the fourth rule of the foregoing chapter, and I find the ſum, or aggregate, to be 437503, equal to the major number, or number from whence the leſſer is ſubtracted. Behold the work in the margin.

437503
153827
———
283676
———
437503

The proof of another example, may be of the firſt example of the ſixth rule of this chapter, where it is required to ſubtract 57 l. 16 s. 3 d. 2 qrs. from 375 l. 13 s. 7 d. 1 qr. and by the rule I find the remainder to be 317 l. 17 s. 3 d. 3 qrs. Now, to prove it, I add the ſaid remainder 317 l. 17 s. 3 d. 3 qrs. to the minor number 57 l. 16 s. 3 d. 2 qrs. and their ſum is 375 l. 13 s. 7 d. 1 qr. equal to the major number, which proves the work to be true; but if it had happened to have been either more or leſs than the ſaid major number, then the operation had been falſe.

l.	*s.*	*d.*	*q.*
375	13	7	1
57	16	3	2
317	17	3	3
375	13	7	1

9. The general effect of ſubtraction, is, to find the difference or exceſs between two numbers; and the reſt, when a payment is made in part of a greater ſum; the date of books printed, the age of any thing, by knowing the preſent year, and the year wherein they were made, created, or built; and ſuch like.

The queſtions appropriated to this rule, are ſuch as follow.

Queſt. 1. What difference is there between one thing of 125 foot long, and another of 66 foot long?

To reſolve this queſtion, I firſt ſet down the major or greater number 125, and under it the minor or leſſer number 66, as is directed in the third rule of this chapter; and, according to the fourth rule of the ſame, I ſubtract the minor from the major; and the remainder, exceſs, or difference, I find to be 59. See the work in the margin.

$$\begin{array}{r}125\\66\\\hline 59\end{array}$$

Queſt. 2. A gentleman oweth a merchant 365 l. whereof he hath paid 278 l. what more doth he owe?

To give an anſwer to this queſtion, I firſt ſet down the major number 365 l. and under it I place 278 the minor, and ſubtract the one from the other; whereby I diſcover the exceſs, difference, or remainder, to be 87; and ſo much is ſtill due to the creditor, as *per* margin.

$$\begin{array}{r}365\\278\\\hline 87\end{array}$$

Queſt. 3. An obligation was written, a book printed, a child born, a church built, or any other thing made in the year of our Lord 1572; and now we account the year of our Lord 1687. The queſtion is to know the age of the ſaid things; that is, how many years are paſſed ſince the ſaid things were made? I ſay, if you ſubtract the leſſer number 1572, from the greater 1687, the remainder will be 115; and ſo many years are paſſed ſince the making of the ſaid things, as by the work in the margin.

$$\begin{array}{r}1687\\1572\\\hline 115\end{array}$$

Queſt. 4. There are three towns that lie in a ſtraight line, *viz.* London, Huntington, and York. Now, the diſtance between the fartheſt of theſe towns, *viz.* London and York, is 151 miles, and from London to Huntington is 49 miles, I demand how far it is from Huntington to York.

To reſolve this queſtion, ſubtract 49, the diſtance between London and Huntington, from 151, the diſtance between London and York, and the remainder is 102 for the true diſtance between Huntington and York. See the work in the margin.

$$\begin{array}{r}151\\49\\\hline 102\end{array}$$

CHAP. VI.

Of multiplication of whole numbers.

1. Multiplication is performed by two numbers of like kind for the production of a third, which ſhall have ſuch reaſon to the one, as the other hath to unit; and, in effect, is a moſt brief and artificial compound addition of many equal numbers of like kind into one ſum. Or, multiplication is that by which we multiply two or more numbers, the one into the other, to the end that their product may come forth, or be diſcovered.

Or, multiplication is the increaſing of any one number by another, ſo often as there are units in that number by which the other is increaſed; or, by having two numbers given, to find a third which ſhall contain one of the numbers as many times as there are units in the other.

2. Multiplication hath three parts. 1*ſt*, The multiplicand, or number to be multiplied. 2*dly*, The multiplier, or number given, by which the multiplicand is to be multiplied. And, 3*dly*, The product or number produced by the other two, the one being multiplied by the other. As if 8 were given to be multiplied by 4, I ſay 4 times 8 is 32; here 8 is the multiplicand, and 4 is the multiplier, and 32 is the product.

$$\begin{array}{r}8\\4\\\hline 32\end{array}$$

3. Multiplication is either ſingle, by one figure; or compound, that conſiſts of many.

Single multiplication is ſaid to conſiſt of one figure, becauſe the multiplicand and multiplier conſiſt each of them of a digit and no more; ſo that the greateſt product that can ariſe by ſingle multiplication, is 81, being the ſquare of 9. And compound multiplication is ſaid to conſiſt of many figures, becauſe the multiplicand or multiplier conſiſts of more places than one: as if I were to multiply 436 by 6, it is called compound, becauſe the multiplicand 436 is of more places than one, *viz.* three places.

4. The learner ought to have all the varieties of ſingle multiplication by heart, before he can well proceed any farther in this art; it being of moſt excellent uſe, and none of the following rules in arithmetic but what have a principal dependence thereupon; which may be learned by the following table.

MULTIPLICATION-TABLE.

1	2	3	4	5	6	7	8	9
2	[illegible]	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
[illegible]	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The uſe of the preceding table is this. In the uppermoſt line or column you have expreſſed all the digits, from 1 to 9; and likewiſe beginning at 1, and going downwards in the ſide-column, you have the ſame; ſo that if you would know the product of any two ſingle numbers multiplied by one another, look for one of them, which you pleaſe, in the uppermoſt column, and for the other in the ſide-column; and running your eye from each figure along the reſpective columns, in the common angle, or place where theſe two columns meet, there is the product required. As for example: I would know how much is 8 times 7. Firſt, I look for 8 in the uppermoſt column, and 7 in the ſide column; then do I caſt my eye from 8, along the column downwards from the ſame, and likewiſe from 7 in the ſide-column, I caſt my eye from thence toward the right hand, and find it to meet with the firſt column at 56; ſo that I conclude 56 to be the product required. It would have been the ſame if you had looked for 7 in the top, and 8 on the ſide. The like is to be underſtood of any other ſuch numbers. The learner being perfect herein, it will be neceſſary to proceed.

5. In compound multiplication, if the multiplicand conſiſts of many places, and the multiplier of but one figure; firſt ſet down the multiplicand, and under it place the multiplier in the place of units, and draw a line underneath them. Then begin, and multiply the multiplier into every particular figure of the multiplicand, beginning at the place of units; and ſo proceed towards the left hand, ſetting each particular product under the line, in order as you proceed: but if any of the products exceed 10, or any number of tens, ſet down the exceſs, and for every 10 carry an unit to be added to the next product; always remembering to ſet down the total product of the laſt figure. Which work being finiſhed, the ſum or number placed under the line ſhall be the true and total product required. As for example: I would multiply 478 by 6. Firſt I ſet down 478, and underneath it 6, in the place of units, and draw a line underneath them, as in the margin. Then I begin, ſaying, 6 times 8 is 48, which is 8 above 4 tens; therefore I ſet down 8, the exceſs, and bear 4 in mind for the 4 tens. Then I proceed, ſaying, 6 times 7 is 42, and four that I carried is 46; I then ſet down 6, and carry 4: and go on, ſaying, 6 times 4 is 24, and 4 that I carried is 28; and becauſe it is the laſt figure, I ſet it all down. And ſo the work is finiſhed; and the product is found to be 2868, as was required.

$$\begin{array}{r}478\\ 6\\ \hline 2868\end{array}$$

6. When in compound multiplication, the multiplier conſiſteth of divers places, then begin with the figure in the place of units in the multiplier, and multiply it into all the figures in the multiplicand, placing the product below the line, as was directed in the laſt example. Then begin with the figure of the ſecond place of the multiplier,

tiplier, (*viz.*) the place of tens, and multiply it likewiſe into the whole multiplicand, as you did the firſt figure, placing its product under the product of the firſt figure. Do in the ſame manner by the third, fourth, and fifth, *&c.* until you have multiplied all the figures of the multiplier particularly into the whole multiplicand, ſtill placing the product of each particular figure under the product of its preceding figure; herein obſerving the following caution.

A caution.

In the placing of the product of each particular figure of the multiplier, you are not to follow the 2d rule of the fourth chapter, *viz.* to place units under units, and tens under tens, *&c.* but to put the figure or cipher in the place of units of the ſecond line under the ſecond figure or place of tens in the line above it, and the figure or cipher in the place of units of the third line, under the place of tens in the ſecond line, *&c.*; obſerving this order till you have finiſhed the work; ſtill placing the firſt figure of every line or product under the ſecond figure, or place of tens in that which is above it. And having ſo done, draw a line under all theſe particular products, and add them together; ſo ſhall the ſum of all theſe products be the total product required.

```
   764
    27
  ----
  5348
 1528
 -----
 20628
```

As if it were required to multiply 764 by 27, I ſet them down the one under the other, with a line drawn underneath them. Then I begin, ſaying, 7 times 4 is 28; then I ſet down 8, and carry 2. Then I ſay, 7 times 6 is 42, and 2 that I carried is 44, that is 4 and go four. Then 7 times 7 is 49, and 4 that I carry is 53, which I ſet down; becauſe I have not another figure to multiply. Thus I have done with the 7. Then I begin with the 2, ſaying, 2 times 4 is 8, which I ſet down under (4) the ſecond figure or place of tens in the line above it, as you may ſee in the margin. Then I proceed, ſaying, 2 times 6 is 12, that is 2, and carry 1. Then 2 times 7 is 14, and 1 that I carry is 15, which I ſet down; becauſe it is the product of the laſt figure: ſo that the product of 764 by 7, is 5348; and by 2, is 1528, which being placed the one under the other, as is before directed, as you ſee in the margin, and a line drawn under them, and they added together reſpectively, make 20628, the true product required, being equal to 27 times 764.

Another example may be this. Let it be required to multiply 5486 by 465: I diſpoſe of the multiplicand and multiplier according to the rule, and begin, multiplying the firſt figure of the multiplier, which is 5, into the whole multiplicand, and find the product is 27430. Then I proceed, and multiply the ſecond figure (6) of the multiplier into the multiplicand, and find the product to amount to 32916; which is ſubſcribed under the other product reſpectively. Then do I multiply the third and laſt figure (4) of the multiplier into the multiplicand, and the product is 21944; which is likewiſe placed under the ſecond line reſpectively. Then I draw a line under the ſaid products, being placed the one under the other according to the rule, and add them together, and the ſum is 2550990, the true product ſought; being equal to 5486 times 465, or 465 times 5486.

5486
465
27430
32916
21944
2550990

More examples in this rule are theſe following.

430865	6400758
4739	37496
3877785	38404548
1292595	57606822
3016055	25603032
1723460	44805306
	19202274
2041869235	240002821968

Compendium in multiplication.

7. Although the former rules are ſufficient for all caſes in multiplication, yet becauſe in the work of multiplication many times great labour may be ſaved, I ſhall acquaint the learner with ſome compendiums in order thereto, *viz.* If the multiplicand or multiplier, or both of them, end with ciphers; then in your multiplying you may neglect

the ciphers, and multiply only the ſignificant figures: and to the product of thoſe ſignificant figures, add ſo many ciphers as the numbers given to be multiplied did end with *; that is, annex them on the right hand of the ſaid product, ſo ſhall that give you the true product required. As if I were to multiply 32000 by 4300, I ſet them down in order to be multiplied, as you ſee in the margin; but neglecting the ciphers in both numbers, I only multiply 32 by 43, and the product I find to be 1376; to which I annex the 5 ciphers that are in the multiplicand and multiplier, and then it makes 137600000 for the true product of 32000 by 4300.

```
    32000
     4300
    -----
      96
    128
---------
137600000
```

8. If in the multiplier, ciphers are placed between ſignificant figures, then multiply only by the ſignificant figures, neglecting the ciphers †. But here ſpecial notice is to be taken of the true placing of the firſt figure after the neglect of ſuch cipher or ciphers: and therefore you muſt obſerve in what place of the multiplier the figure you multiply by ſtandeth, and ſet the firſt figure of that product under the ſame place of the product of the firſt figure of your multiplier. As for example: Let it be required to multiply 371568 by 40007. Firſt I multiply the multiplicand by 7, and the product is 2600976. Then, neglecting the ciphers, I multiply by 4, and that product is 1486272. Now I conſider that 4 is the 5th figure in the multiplier; therefore I place 2 (the firſt figure of the product by 4) under the 5th place of the 1ſt product by 7, and the reſt in order: and having added them together, the total product is found to be 14865320976. Other examples in this rule are theſe following.

```
     371568
      40007
-----------
    2600976
1486272
-----------
14865320976
```

---

* *Si ex numeris propoſitis unus vel uterque adjunctos habeat ad dexteram circulos, omiſſis circulis, fiat ipſorum numerorum multiplicatio, et facto demum tot inſuper integrorum, loci, accenſeantur quot ſunt omiſſi circuli in utroque factore.* Clavis Mat. chap. iv. 3.

† *Si in medio multiplicantis loco circulus fuerit, ille negligitur.* Alſted. chap. vi. de arith.

```
   327586
     6030
  -------
   982758
 1965516
 ----------
 1975343580
```

```
     7864371
       20604
    --------
    31457484
  47186226
 15728742
 ------------
 162037500084
```

9. If you are to multiply any number by an unit with ciphers (*viz.*) by 10, 100, 1000, *&c.* then annex ſo many ciphers before the multiplicand, and the number, when the ciphers are annexed, is the product required. As if you would multiply 428 by 100, annex two ciphers to 428, and and it is 42800. If it were required to multiply 100 by 10000, annex 4 ciphers, and it gives 1000000 for the product required.

The proof of multiplication.

10. Multiplication is proved by diviſion; and, to ſpeak truth, all other ways are falſe *: and therefore it will be moſt convenient, in the firſt place, to learn diviſion, and by that to prove multiplication. There is a way (at this day generally uſed in ſchools) to prove multiplication, which is this. Firſt add all the figures in the multiplicand together, as if they were ſimple numbers, caſting away the nines as oft as it comes to ſo much, noting the remainder at laſt, which in this caſe cannot be ſo much as 9: caſt likewiſe the nines out of the multiplier as you did out of the multiplicand, and note the remainder. Then multiply the remainders, one by the other, and caſt the nines out of the product, obſerving the remainder. And, laſtly, caſt the nines out of the total product, and if this remainder be equal to the remainder laſt found, then they conclude the work to be rightly performed. But there may be given a thouſand, nay infinite falſe products in multiplication, which after this manner may be proved to be true: and therefore this way of proving doth not deſerve any example. But we ſhall defer the proof of

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* *Namque non eſt quod aliam expectes examinandi viam; nam aliæ vulgares et falſæ ſunt, et nullo innixæ fundamento.* Gemma Friſius.

this rule till we come to prove diviſion, and then we ſhall prove them both together.

11. The general effect of multiplication is contained in the definition of the ſame; which is, to find out a third number, ſo often containing one of the two given numbers, as the other containeth unity.

The ſecond effect is, by having the length and breadth of any thing, (as a parallelogram or long plane), to find the ſuperficial contents of the ſame; and by having the ſuperficial content of the baſe, and the length, to find out the ſolidity of any parallelopipedon, cylinder, or other ſolid figures.

The third effect is, by the contents, price, value, buying, ſelling, expenſe, wages, exchange, ſimple intereſt, gain or loſs of any one thing, be it money, merchandiſe, *&c.* to find out the value, price, expenſe, buying, ſelling, exchange, or intereſt of any number of things of like name, nature, and kind.

The fourth effect is not much unlike the other; by the contents, value, or price of any one part of any thing denominated, to find out the contents, value, or price of the whole thing, all the parts into which the whole is divided, multiplying the price of one of thoſe parts.

The fifth effect is, to aid, to compound, and to make other rules; as chiefly the rule of proportion, called the *golden rule* or *rule of three.* Alſo by it, things of one denomination are reduced to another.

If you multiply any number of integers, or the price of the integer, the product will diſcover the price of the quantity, or number of integers given.

In a rectangular ſolid, if you multiply the breadth of the baſe by the depth, and that product by the length, this laſt product will diſcover the ſolidity or content of the ſame ſolid.

Some queſtions proper to this rule, may be theſe following.

Queſt. 1. What is the content of a ſquare piece of ground, whoſe length is 28 perches, and breadth 13 perches?

Anſwer. 364 ſquare perches: for multiplying 28 the length by 13 the breadth, the product is ſo much.

Queſt. 2. There is a ſquare battle, whoſe flank is 47 men, and the files 19 deep; what number of men doth that battle contain? *Facit* 893: for multiplying 47 by 19, the product is 893.

Queſt. 3. If any one thing coſt 4 ſhillings, what ſhall 9 things coſt? *Anſ.* 36 ſhillings: for multiplying 4 by 9, the product is 36.

Queſt. 4. If a piece of money or merchandiſe be worth or coſt 17 ſhillings, what ſhall 19 ſuch pieces of money or merchandiſe coſt? *Facit* 323 ſhillings, which is equal to 16 l. 3 s.

Queſt. 5. If a ſoldier or ſervant get or ſpend 14 s. *per* month, what is the wages or charges of 49 ſoldiers or ſervants for the ſame time? Multiply 49 by 14, the product is 686 s. or 34 l. 6 s. for the anſwer.

Queſt. 6. If in a day there are 24 hours, how many hours are there in a year, accounting 365 days to conſtitute the year? *Facit* 8760 hours: to which if you add the 6 hours over and above 365 days, as there is in a year, then it will be 8766 hours. Now, if you multiply this 8766 by 60, the number of minutes in an hour, it will produce 525960, the number of minutes in a year.

CHAP. VII.

Of diviſion of whole numbers.

1. Diviſion is the ſeparating or parting of any number or quantity given into any parts aſſigned; or to find how often one number is contained in another; or from any two numbers given, to find a third that ſhall conſiſt of ſo many units, as the one of thoſe two given numbers is comprehended or contained in the other.

2. Diviſion hath three parts or numbers remarkable, *viz.* firſt, the dividend; ſecondly, the diviſor; thirdly, the quotient. The dividend is the number given to be parted or divided. The diviſor is the number given by which the dividend is divided; or it is the number which ſheweth how many parts the dividend is to be divided into. And the quotient is the number produced by

the diviſion of the two given numbers, the one by the other.

So 12 being given to be divided by 3, or into three equal parts, the quotient will be 4; for 3 is contained in 12 four times: where 12 is the dividend, and 3 is the diviſor, and four is the quotient.

3. In diviſion ſet down your dividend, and draw a crooked line at each end of it; and before the line at the left hand place the diviſor, and behind that on the right hand place the figures of the quotient, as in the margin; where it is required to divide 12 by 3. Firſt I ſet down 12 the dividend, and on each ſide of it do I draw a crooked line, and before that on the left hand do I place 3 the diviſor. Then do I ſeek how often 3 is contained in 12; and becauſe I find it 4 times, I put 4 behind the crooked line on the right hand of the dividend, denoting the quotient.

3) 12 (4

4. But if, when the diviſor is a ſingle figure, the dividend conſiſteth of two or more places; then, having placed them for the work, as is before directed, put a point under the firſt figure on the left hand of the dividend, provided it be bigger than, or equal to the diviſor; but if it be leſs than the diviſor, then put a point under the ſecond figure from the left hand of the dividend: which figures, as far as the point goeth from the left hand, are to be reckoned by themſelves as if they had no dependence upon the other part of the dividend, and, for diſtinction's ſake, may be called the *dividual*. Then aſk how often the diviſor is contained in the dividual; placing the anſwer in the quotient. Then multiply the diviſor by the figure that you placed in the quotient, and ſet the product thereof under your dividual. Then draw a line under the product, and ſubtract the ſaid product from the dividual, placing the remainder under the ſaid line. Then put a point under the next figure in the dividend on the right hand of that to which you put the point before, and draw it down, placing it on the right hand of the remainder which you found by ſubtraction; which remainder, with the ſaid figure annexed to it, ſhall be a new dividual. Then ſeek again how often the diviſor is contained in this new dividual; and put the anſwer in

the quotient on the right hand of the figure which you put there before. Then multiply the diviſor by the laſt figure that you put in the quotient, and ſubſcribe the product under the dividual, and make ſubtraction; and to the remainder draw down the next figure from the grand dividend, (having firſt put a point under it), and put it on the right hand of the remainder for a new dividual, as before, &c. and proceed thus till the work is finiſhed.

Obſerving this general rule in all kinds of diviſion.

Firſt, To ſeek how often the diviſor is contained in the dividual. Then, having put the anſwer in the quotient, multiply the diviſor thereby, and ſubtract the product from the dividual. An example or two will make the rule plain. Let it be required to divide 2184 by 6. I diſpoſe the numbers given as is before directed, and as you ſee in the margin, in order to the work.

```
6) 2184 (3
    .
```

Then, becauſe 6 the diviſor is more than 2 the firſt figure of the dividend, I put a point under 1 the ſecond figure, which makes 21 for the dividual. Then do I aſk how often 6 the diviſor is contained in 21; and becauſe I cannot have it more than 3 times, I put 3 in the quotient; and thereby do I multiply the diviſor (6), and the product is 18; which I ſet in order under the dividual, and ſubtract it therefrom, and the remainder (3) I place in order under the line, as you ſee in the margin.

```
6) 2184 (3
    .
   18
   ---
    3
```

Then do I make a point under the next figure of the dividend, being 8, and draw it down, annexing it to the remainder 3; ſo have I 38 for a new dividual. Then do I ſeek how often 6 is contained in 38; and becauſe I cannot have it more than 6 times, I put 6 in the quotient; and thereby do I multiply the diviſor (6), and the product (36) I put under the dividual (38), and ſubtract it therefrom, and the remainder (2) I put under the line, as you ſee in the margin.

```
6) 2184 (36
    ..
   18
   ---
    38
    36
    ---
     2
```

Then do I put a point under the next (and laſt) figure of the dividend, (being 4), and draw it down to the remainder (2); and putting it on the right hand thereof, it maketh 24 for a new dividual. Then I ſeek how often 6 is contained in 24; and the anſwer is 4; which I put in the quotient, and multiply the diviſor (6) thereby; and the product (24) I put under the dividual (24), and ſubtract it therefrom, and the remainder is 0. And thus the work is finiſhed; and I find the quotient to be 364; that is, 6 is contained in 2184 juſt 364 times, or 2184 being divided into 6 equal parts, 364 is one of thoſe parts.

```
6) 2184 (364
   ...
   18
   ---
    38
    36
    ---
     24
     24
     ---
     (0)
```

Again, if it were required to divide 2646 by 7, or into 7 equal parts, the quotient will be found to be 378; as appeareth by the operation on the margin.

```
7) 2646 (378
   ...
   21
   ---
    54
    49
    ---
     56
     56
     ---
     (0)
```

So if it were required to divide 946 by 8, the quotient will be found to be 118, and 2 remaining after diviſion is ended. The work appeareth on the margin.

```
8) 946 (118
   ...
   8
   ---
   14
    8
   ---
    66
    64
   ---
   (2)
```

Many times the dividend cannot exactly be divided by the diviſor, but ſomething will remain; as in the laſt example; where 946 was given to be divided by 8, the quotient was 118, and there remaineth 2 after the diviſion is ended. Now, what is to be done in this caſe with the remainder, the learner ſhall be taught when we come to treat of the reducing (or reduction) of fractions.

And here note, that if, after your diviſion is ended, any thing do remain, it muſt be leſs than your diviſor; for otherwiſe your work is not rightly performed.

Other examples are ſuch as follow.

8) 73464 (9183
72
14
8
66
64
24
24
(0)

9) 13758 (1528
9
47
45
25
18
78
72
(6)

5. But if the diviſor conſiſteth of more places than one, then chuſe ſo many figures from the left ſide of the dividend for a dividual as there are figures in the diviſor; and put a point under the fartheſt figure of that dividual to the right hand, and ſeek how often the firſt figure on the left ſide of the diviſor is contained in the firſt figure on the left ſide of the dividual, and place the anſwer in the quotient, and thereby multiply your diviſor, placing your product under your dividual, and ſubtract it therefrom, placing the remainder below the line. Then put a point under the next figure in the dividend, and draw it down to the ſaid remainder, and annex it on the right ſide thereof, which makes a new dividual; and proceed as before till the work is finiſhed.

And if it ſo happen, that, after you have choſen your firſt dividual, as is before directed, you find it to be leſs than the diviſor; then put a point under the figure more near to the right hand, and ſeek how often the firſt figure on the left ſide of the diviſor is contained in the two firſt figures on the left ſide of the dividual, and place the anſwer in the quotient; by which multiply the diviſor, and place the product thereof in order under the dividual, and ſubtract it therefrom, and then proceed as before.

Always remembering, that in all caſes of diviſion, if, after you have multiplied your diviſor by the figure firſt placed in the quotient, the product be greater than the dividual, then you muſt cancel that figure in the quotient, and, inſtead thereof, put a figure leſs by an unit, (or one), and multiply the diviſor thereby; and, if ſtill the product be greater than the dividual, make the figure in the quotient yet leſs by an unit. And thus do, until your product be leſs than the dividual, or at the moſt equal thereto, and then make ſubtraction, *&c.*

So, if you would divide 9464 by 24, the quotient will be found to be 394. I firſt put down the given number, as is before directed in the third rule. Now, becauſe my diviſor conſiſteth of two figures, I therefore put a point under the ſecond figure from the left hand of my dividend, which here is 4; wherefore I ſeek how often 2 (the firſt figure on the left ſide of the diviſor) is contained in 9. (the like firſt in the dividual); the anſwer is 4; which I put in the quotient, and thereby multiply all the diviſor, and find the product to be 96, which is greater than the dividual 94: wherefore I cancel the 4 in the quotient, and, inſtead thereof, I put 3, (an unit leſs), and by it multiply the diviſor 24. and the product is 72; which I ſubtract from 94 the dividual, and the remainder is 22. Then do I make a point under the next figure 6 in the dividend, and draw it down, and place it on the right ſide of the remainder 22, and it makes 226 for a new dividual. Now, becauſe the dividual 226 conſiſteth of a

```
              (3
24) 9464     (
     ...      (4
    72
    ---
    22
```

figure more than the diviſor, therefore I ſeek how often 2 (the firſt figure of the diviſor) is contained in 22, the two firſt of the dividual: I ſay 9 times: wherefore I put 9 in the quotient, and thereby multiply the diviſor 24; the product (216) I place under the dividual 226, and ſubtract it from it, and there remaineth 10.

```
24) 9464 (39
     ..
    72
    ---
    226
    216
    ---
     10
```

Then I go on, and make a point under the next and laſt figure (4) in the dividend, and draw it down to the remainder 10, and it makes 104 for a new dividual: which is alſo a figure more than the diviſor; and therefore I ſeek how often 2 is contained in 10: I anſwer 5 times. But multiplying my diviſor by 5, the product is 120; which is greater than the dividual: and therefore I make it but 4; and by it multiply the diviſor; and the product is 96; which being placed under, and ſubtracted from the dividual, there remaineth 8. And thus the whole work of this diviſion is ended; and I find, that 9464 being divided by 24, or into 24 equal parts, is found to be 394, as was ſaid before, and the remainder is 8; as you ſee in the work on the margin.

```
24) 9464 (394
     ...
    72
    ---
    226
    216
    ---
     104
      96
     ---
     (8)
```

Another example may be this. Let there be required the quotient of 1183653 divided by 385. Firſt I diſpoſe of the numbers, in order to their dividing; and becauſe 118, the three firſt figures of the dividend, is leſs than the diviſor 385, I therefore make a point under the fourth figure, which is 3, and ſee how often 3 (the firſt figure of the diviſor) is contained in 11: the anſwer is 3; which I put in the quotient, and thereby multiply the diviſor 385, and the product is 1155; which I ſubtract from the dividual 1183, and there remains 28. Then (as before) I draw down the next figure, which is 6, and place it before the remainder

```
385) 1183653 (3
        .
     1155
     ----
       28
```

mainder 28: ſo have I 268 for a new dividual; and becauſe it hath no more figures than the diviſor, I ſeek how often 3 (the firſt figure in the diviſor) is contained in 2, (the firſt figure of the dividual), and the anſwer is 0; for a greater number cannot be contained in a leſſer; wherefore I put 0 in the quotient: and thereby (according to the 5th rule) I ſhould multiply my diviſor; but if I do, the product will be 0; and 0 ſubtracted from the dividual 286, the remainder is the ſame.

```
385) 1183653 (30
       ..
     1155
     ----
      286
```

Wherefore I draw down the next figure (5) from the dividend, and put it before the ſaid remainder 286: ſo have I 2865 for a new dividual; and becauſe it conſiſteth of four places, *viz.* a place more than the diviſor, I ſeek how often 3 (the firſt figure of the diviſor) is contained in 28 (the two firſt of the dividual), and I ſay, there is 9 times 3 in 28; but multiplying my whole diviſor (385) thereby, I find the product to be 3465, which is greater than the dividual 2865: wherefore I chuſe 8, which is leſs by an unit than 9; and thereby I multiply my diviſor 385, and the product is 3080; which ſtill is greater than the ſaid dividual: wherefore I chuſe another number yet an unit leſs, *viz.* 7, and having multiplied my diviſor thereby, the product is 2695; which is leſs than the dividual 2865: wherefore I put 7 in the quotient, and ſubtract 2695 from the dividual 2865, and there remains 170.

```
385) 1183653 (307
       ...
     1155
     ----
      2865
      2695
      ----
       170
```

Then I draw down the laſt figure (3) in the dividend, and place it before the ſaid remainder 170, and it makes 1703 for a new dividual. Then (for the reaſon above ſaid) I ſeek how often 3 is contained in 17: the anſwer is 5; but multiplying the diviſor thereby, the product is 1925, greater than the dividual: wherefore I ſay it will bear 4, an unit leſs,

```
385) 1183653 (3074
       ....
     1155
     ----
      2865
      2695
      ----
       1703
       1540
       ----
       (163)
```

and by it I multiply the diviſor 385; and the product is 1540, which is leſs than the dividual; and therefore I put 4 in the quotient, and ſubtract the ſaid product from the dividual, and there remaineth 163. And thus the work is finiſhed; and I find that 1183653 being divided by 385, or into 385 equal ſhares or parts, the quotient or one of thoſe parts, is 3074; and beſides there is 163 remaining.

And thus the learner being well verſed in the method of the foregoing examples, he may be ſufficiently qualified for the dividing of any greater ſum or number, into as many parts as he pleaſeth; that is, he may underſtand the method of dividing by a diviſor which conſiſteth of 4, or 5, or 6, or any greater number of places; the method being the ſame with the foregoing examples in every reſpect.

Other examples in diviſion.

27986) 835684790 (29860	196374) 473986018 (2413
55972	392748
275964	812380
251874	785496
240907	268841
223888	196374
170199	724678
167916	589122
Remains (22830)	Remains (135556)

So if you divide 47386473 by 58736, you will find the quotient to be 806, and 45257 will remain after the work is ended.

In like manner, if you would divide 3846739204 by 483064, the quotient will be 7963, and the remainder after diviſion will be 100572.

Compendiums in diviſion.

1. IF any given number be to be divided by another number that hath ciphers annexed on the right ſide thereof; omitting the ciphers, you may cut off ſo many figures from the right hand of the dividend, as there are ciphers before the diviſor, and let the remaining numbers in the dividend be divided by the remaining number or numbers of the diviſor; obſerving this caution, that if after your diviſion is ended any thing remain, you are to annex thereto the number or numbers that were cut off from the dividend[*], and ſuch new found number ſhall be the remainder. As for example: Let it be required to divide 46658 by 400. Now, becauſe there are two ciphers before the diviſor, I cut off as many figures from before the dividend, *viz.* 58; ſo that then there will remain only 466 to be divided by 4, and the quotient will be 116, and there will remain 2; to which I annex the two figures (58) which were cut off from the dividend, and it makes 258 for the true remainder: ſo that I conclude 46658 being divided by 400, the quotient will be 116, and 258 remains after the work is ended; as by the work in the margin.

4|00) 466|58 (116
. . .
4
--
6
4
--
26
24
---
(258)

2. And hence it followeth,[†] that if the diviſor be 1, or an unit with ciphers annexed, you may cut off ſo many figures from the dividend, as there are ciphers in the diviſor; and then the figure or figures that are on the left

---

[*] *Et ſi diviſor adjunctos ſibi habeat circulos ad dextram; omiſſis circulis, et abſciſſis totidem ultimis figuris dividendi, in numeris reliquis fiat diviſio; in fine autem diviſionis reſtituendi ſunt tum omiſſi circuli tum figuræ abſciſſæ.* Ough. Clav. Math. cap. 5. 3.

[†] *Diviſurus quemcunque numerum per* 10, *aufer ex dextra parte unicam eamque primam figuram: reliquæ enim figuræ productum oſtendunt; ablatum reſiduum, &c.* Gem. Friſ. arith. pars 1.

hand will be the quotient, and thoſe that are on the right hand will be the remainder after the diviſion is ended. As, thus: If 45783 were to be divided by 10, I cut off the laſt figure (3) with a daſh thus, (4578|3), and the work is done; and the quotient is 4578, (the number on the left hand of the daſh), and the remainder is 3 (on the right hand). In like manner, if the ſame number 45783 were to be divided by 100, I cut off two figures from the end thus, (457|83); and the quotient is 457; and the remainder is 83. And if I were to divide the ſame by 1000, I cut off three figures from the end thus, (45|783); and the quotient is 45, and 783 the remainder, *&c.*

3. The general effect of diviſion is contained in the definition of the ſame; that is, by having two unequal numbers given, to find a third number in ſuch proportion to the dividend, as the diviſor hath to unit or 1. It alſo diſcovers what reaſon or proportion there is between numbers: ſo if you divide 12 by 4, it quotes 3; which ſhews the reaſon or proportion of 4 to 12 is triple.

The ſecond effect is, by the ſuperficial meaſure or content, and the length of any oblong, rectangular parallelogram, or ſquare plane, known, to find out the breadth thereby; or contrariwiſe, by having the ſuperficies and breadth of the ſaid figure, to find out the length thereof: alſo, by having the ſolidity and length of a ſolid, to find the ſuperficies of the baſe; *& contra.*

The third effect is, by the contents, reaſon, price, value, buying, ſelling, expenſes, wages, exchange, intereſt, profit or loſs of any number of things, (be it money, merchandiſe, or what elſe), to find out the contents, reaſon, price, value, buying, ſelling, expenſe, wages, exchange, intereſt, profit, or loſs, of any one thing of like kind.

The fourth effect is, to aid, to compoſe, and to make other rules; but principally the rule of proportion, called the *golden rule*, or *rule of three*; and the reduction of monies, weights, and meaſures, of one denomination, into another. By it alſo fractions are abbreviated, by finding a common meaſurer unto the numerator and denominator, thereby diſcovering commenſurable numbers.

If you divide the value of any certain quantity by the ſame quantity, the quotient diſcovers the rate or value of the integer. As if 8 yards of cloth coſt 96 ſhillings ; if you divide 96, the value or price of the given quantity, by 8, the quotient will be 12 s. which is the price or value of 1 of thoſe yards ; *& contra.*

If you divide the value or price of any unknown quantity by the value of the integer, it gives you in the quotient that unknown quantity whoſe price is thus divided. As, if 12 ſhillings were the value of 1 yard, I would know how many yards are worth 96 ſhillings. Here, if you divide 96, the price or value of the unknown quantity, by 12, the rate of the integer, or 1 yard, the quotient will be 8; which is the number of yards worth 96 ſhillings.

Some queſtions anſwered by diviſion, may be theſe following.

Queſt. 1. If 22 things coſt 66 ſhillings, what will 1 ſuch thing coſt ? *Facit* 3 ſhillings: for if you divide 66 by 22, the quotient is 3 for the anſwer So if 36 yards or ells of any thing be bought or ſold for 108 l. how much will 1 yard or ell be bought or ſold for ? *Facit* 3 l. for if you divide 108 l. by 36 yards, the quotient will be 3 l. the price of the integer.

Queſt. 2. If the expenſe, charges, or wages of 7 years amount to 868 l. what is the expenſe, charges, or wages of 1 year ? *Facit* 124 l. for if you divide 868 (the wages of 7 years) by 7 (the number of years), the quotient will be 124 l. for the anſwer. See the work.

7) 868 (124
. . .
7
16
14
28
28
(0)

Queſt 3. If the content of one ſuperficial foot be 144 inches, and the breadth of a board be 9 inches, how many inches of that board in length will make ſuch a foot ? *Facit* 16 inches. for by dividing 144 (the number of ſquare inches

9) 144 (16 inches,
. .
9
54
54
(0)

in a ſquare foot) by 9 (the inches in the breadth of the board), the quotient is 16 for the number of inches in length of that board to make a ſuperficial foot.

Queſt. 4. If the content of an acre of ground be 160 ſquare perches, and the length of a furlong (propounded) be 80 perches, how many perches will there go in breadth to make an acre? *Facit* 2 perches; for if you divide 160 (the number of perches in an acre) by 80 (the length of the furlong in perches), the quotient is 2 perches; and ſo many in breadth of that furlong will make an acre.

```
80) 160 (2 perches
     .
    160
    ---
    (0)
```

Queſt. 5. If there be 893 men to be made up into a battle, the front conſiſting of 47 men, what number muſt there be in the file? *Facit* 19 deep in the file; for if you divide 893 (the number of men) by 47 (the number in the front), the quotient will be 19 in depth of the file. See the work in the margin.

```
47) 893 (19 deep in file.
    ..
    47
    ---
    423
    423
    ---
    (0)
```

Queſt. 6. There is a table, whoſe ſuperficial content is 72 feet, and the breadth of it at the end is 3 feet; now I demand what is the length of this table? *Facit* 24 feet long; for if you divide 72 (the content of the table in feet) by 3 (the breadth of it), the quotient is 24 feet for the length thereof, which was required. See the operation in the margin.

```
3) 72 (24
   ..
   6
   --
   12
   12
   --
   (0)
```

The proof of multiplication and diviſion.

Multiplication and diviſion interchangeably prove each other; for if you would prove a ſum in diviſion, whether the operation be right or no, multiply the quotient by the diviſor; and if any thing remain after the diviſion is ended, add it to the product, which product, if your ſum was rightly divided, will be equal to the dividend. And contrariwiſe, if you would prove a ſum in multiplication, divide the product by the multiplier, and if the work was rightly performed, the quotient will be equal to the multiplicand. See the example where the work is done and undone. Let 7654 be given to be multiplied by 3242, the product will be 24814268, as by the work appeareth.

```
    7654
    3242
 --------
   15308
  30616
 15308
22962
--------
24814268
```

And then if you divide the ſaid product 24814268 by 3242 the multiplier, the quotient will be 7654, equal to the given multiplicand.

```
3242) 24814268 (7654
      ....
      22694
      -----
       21202
       19452
       -----
        17506
        16210
        -----
         12968
         12968
         -----
          (0)
```

In like manner, (to prove a ſum or number in diviſion), if 24814268 were divided by 3242, the quotient will be found to be 7654. Then, for proof, if you multiply 7654 the quotient, by 3242 the diviſor, the product will amount to 24814268, equal to the dividend.

```
          7654
          3242
         -----
         15308
        30616
       15308
      22962
7654) 24814268 (3242
      ...
      22962
      -----
       18522
       15308
       -----
        32146
        30616
        -----
         15308
         15308
         -----
          (0)
```

Or you may prove the laſt, or any other example in multiplication, thus, *viz.* Divide the product by the multiplicand, and the quotient will be equal to the multiplier. See the work.

From whence there ariſeth this corollary, That any operation in diviſion may be proved by diviſion: for if after your diviſion is ended, you divide the dividend by the quotient, the new quotient thence ariſing will be equal to the diviſor of the firſt operation; for trial whereof, let the laſt example be again repeated.

```
3242) 24814268 (7654
      · · · ·
      22694
       21202
       19452
        17506
        16210
         12968
         12968
          (0)
```

For proof whereof, divide again 24814268 by the quotient 7654, and the quotient hence will be equal to the firſt diviſor 3242. See the work.

```
7654) 24814268 (3242
      · · · ·
      22962
       18522
       15308
        32146
        30616
         15308
         15308
          (0)
```

But in proving diviſion by diviſion, the learner is to obſerve this following caution, That if after his diviſion is ended, there be any remainder; before you go about to prove your work, ſubtract that remainder out of your dividend, and then work as in the following example, where it is required to divide 43876 by 765, the quotient here is 57, and the remainder is 271. See the work in the margin.

```
765) 43876 (57
     · ·
     3825
      5626
      5355
      (271)
```

Now, to prove this work, ſubtract the remainder 271 out of the dividend 43876, and there remaineth 43605 for a new dividend to be divided by the former quotient 57, and the quotient thence ariſing is 765, equal to the given diviſor, which proveth the operation to be right.

```
      43876
        271
     ------
57)  43605 (765
     ...
     399
     ---
      370
      342
      ---
       285
       285
       ---
       (0)
```

Thus have we gone through the four ſpecies of arithmetic, *viz.* addition, ſubtraction, multiplication, and diviſion, upon which the following rules, and all other operations whatſoever that are poſſible to be wrought by numbers, have their immediate dependence, and by them are reſolved. Therefore before the learner make a farther ſtep in this art, let him be well acquainted with what hath been delivered in the foregoing chapters *.

C H A P. VIII.

Of Reduction.

1. Reduction is that which brings together two or more numbers of different denominations into one denomination; or it ſerveth to change or alter numbers, money, weight, meaſure, or time, from one denomination to another; and likewiſe to abridge fractions to the loweſt terms: all which it doth ſo preciſely, that the firſt proportion remaineth without the leaſt jot of error or wrong committed: ſo that it belongeth as well to fractions as integers; of which in its proper place. Reduction is

---

* *Hæ ſunt igitur quatuor illæ ſpecies arithmetices per quas omnia quæcunque deinceps dicenda ſunt, vel quæ per numeros fieri poſſibile eſt, abſolvuntur; quare eas quiſquis et cito omnia perdiſcito.* Gem. Friſ. arithm. par. I.

generally performed either by multiplication or diviſion. From whence we may gather, That,

2. Reduction is either deſcending or aſcending.

3. Reduction deſcending is, when it is required to reduce a ſum or number of a greater denomination into a leſſer, which number, when it is ſo reduced, ſhall be equal in value to the number firſt given in the greater denomination: as if it were required to know how many ſhillings, pence, or farthings, are equal in value to an hundred pounds? or, how many ounces are contained in 45 hundred weight? or, how many days, hours, or minutes, there are in 240 years? *&c.* And this kind of reduction is generally performed by multiplication.

4. Reduction aſcending is, when it is required to reduce or bring a ſum or number of a ſmaller denomination into a greater, which ſhall be equivalent to the given number: as ſuppoſe it were required to find out how many pence, ſhillings, or pounds, are equal in value to 45785 farthings? or, how many hundreds are equal to (or in) 3748 pounds, *&c.* And this kind of reduction is always performed by diviſion.

5. When any ſum or number is given to be reduced into another denomination, you are to conſider whether it ought to be reſolved by the rule decending or aſcending, *viz.* by multiplication, or diviſion. If it be to be performed by multiplication, conſider how many parts of the denomination into which you would reduce it are contained in an unit or integer of the given number, and multiply the ſaid given number thereby, and the product thereof will be the anſwer to the queſtion. As, if the queſtion were, In 38 pounds how many ſhillings? Here I conſider, that in 1 pound are 20 ſhillings, and that the number of ſhillings in 38 pounds, will be 20 times 38; wherefore I multiply 38 l. by 20, and the product is 760, and ſo many ſhillings are contained in 38 pounds, as in the margin.

$$\begin{array}{r}38\\20\\\hline 760\end{array}$$

But when there is a denomination or denominations between the number given and the number required, you may, if you pleaſe, reduce it into the next inferior denomination, and then into the next lower than that, *&c.*

until you have brought it into the denomination required. As for example: Let it be demanded, In 132 pounds how many farthings? Firſt, I multiply 132 (the number of pounds given) by 20 to bring it into ſhillings, and it makes 2640 ſhillings. Then do I multiply the ſhillings 2640 by 12, to bring them into pence, and it produceth 31680, and ſo many pence are contained in 2640 ſhillings, or 132 pounds. Then do I multiply the pence, *viz.* 31680 by 4 to bring them into farthings, (becauſe 4 farthings is a penny), and I find the product thereof to be 126720, and ſo many farthings are equal in value to 132 pounds. The work is manifeſt in the margin.

```
   132 pounds
    20
  ------
  2640 ſhillings
    12
  ------
   528
  264
  -------
  31680 pence
      4
  -------
 126720 farth.
```

6. And if the number propounded to be reduced is to be divided, or wrought by the rule aſcending, conſider how many of the given numbers are equal to an unit or integer in that denomination to which you would reduce your given number, and make that your diviſor, and the given number your dividend; and the quotient thence ariſing will be the number ſought or required. As for example: Let it be required to reduce 2640 ſhillings into pounds. Here I conſider that 20 ſhillings are equal to 1 pound; wherefore I divide 2640 (the given number) by 20, and the quotient is 132, and ſo many pounds are contained in 2640 ſhillings. In reduction deſcending and aſcending, the learner is adviſed to take particular notice of the tables delivered in the ſecond chapter of this book, where he may be informed what multipliers or diviſors to make uſe of in the reducing of any number to any other denomination whatſoever, eſpecially Engliſh monies, weights, meaſures, time, and motion.

```
                l.
2|0) 264|0 (132
      ...
       2
       —
       6
       6
       —
       4
       4
       —
      (0)
```

But in this place it is not convenient to meddle with foreign coins, weights, or meaſures.

But if in reduction aſcending it happen that there is a denomination or denominations between the number given and the number required, then you may reduce your number given into the next ſuperior denomination; and when it is ſo reduced, bring it into the next above that; and ſo on, until you have brought it into the denomination required. As for example:

Let it be demanded, In 126720 farthings how many pounds? Firſt I divide my given number (being farthings) by 4, to bring them into pence, (becauſe 4 farthings make one penny); and there are 31680 pence. Then I divide 31680 pence by 12, and the quotient giveth 2640 ſhillings. And then I divide 2640 ſhillings by 20, and the quotient giveth 132 pounds; which are equal in value to 126720 farthings. See the whole work as it followeth.

```
                12)      2|0)    l.
4) 126720  (31680  (264|0  (132
    .....    ....      ..

   12        24       2
   --       ---       -
    6        76       6
    4        72       6
   --       ---      ---
   27        48       4
   24        48       4
  ---       ---      ---
    32      (0)      (0)
    32
    --
   (0)
```

7. When the number given to be reduced, conſiſteth of divers denominations, as pounds, ſhillings, pence, and farthings; or of hundreds, quarters, pounds, and ounces, &c. then you are to reduce the higheſt (or greateſt) denomination into the next inferior, and add thereunto the number ſtanding in that denomination, which your greateſt or higheſt number is reduced to. Then reduce that ſum into the next inferior denomination, adding thereto the number ſtanding in that denomination. Do ſo until you have brought the number given

into the denomination propoſed. As, if it were required to reduce 48 l. 13 s. 10 d. into pence: firſt I bring 48 l. into ſhillings, by multiplying it by 20, and the product is 960 ſhillings; to which I add the 13 ſhillings, and they make 973. Then I multiply 973 by 12, to bring the ſhillings into pence, and they make 11676 pence; to which I add the 10 pence, and they make 11686 pence for the anſwer. See the work done.

```
       l.   s.   d.
      48—13—10
       20
      ———
      960 ſhillings
Add    13
      ———
Sum   973 ſhillings
       12
      ———
     1946
     973
    ————
    11676 pence
Add    10
    ————
Sum 11686 pence
```

8. If in reduction aſcending, after diviſion is ended, any thing remain, ſuch remainder is of the ſame denomination with the dividend.

Example. In 4783 farthings, I demand how many pounds? View the following operation.

First, I divide the given number of farthings (*viz.* 4783) by 4, to bring them into pence, and the quotient is 1195 pence; and there remaineth 3 after the work of diviſion is ended, which is 3 farthings.

```
            12)    2|0)
4) 4783   (1195   (9|9  (4 pounds
   ....     ..
   4        108    8
   —        ———    —
   7         115  (19) ſhillings
   4         108
   —        ———
    38  Rem. (7) pence
    36
    —               l.   s.  d.  q.
     23      Facit  4—19—7—3
     20
     —
Rem. (3) farthings
```

Again, I divide 1195 pence (the ſaid quotient) by 12, to reduce them into ſhillings, and the quotient is 99 ſhillings; and there is a remainder of 7, which is 7 pence.

And then divide 99 ſhillings (the laſt quotient) by

20, to bring it into pounds, and the quotient is 4 l. and there remaineth 19 ſhillings: ſo that I conclude that in 4783 (the propoſed number of farthings) there is 4 pounds, 19 ſhillings, 7 pence, 3 farthings.

More examples in reduction of coin.

Queſt. 1. In 438 l. how many ſhillings? *Facit* 8760 ſhillings; for multiplying 438 by 20, the product amounteth to ſo much. See the work.

438 pounds
20
Facit 8760 ſhillings

Queſt. 2. In 467 l. how many pence? Firſt multiply the given number of pounds (467) by 20, to bring it into ſhillings, and it makes 9340 ſhillings; then multiply the ſhillings by 12, and it produceth 112080 pence, thus.

467 pounds
20
9340 ſhill.
12
1868
934
Facit 112080 pence

Or it may be reſolved thus, *viz.* Multiply the given number of pounds (467) by (240) the number of pence in a pound, and the product is the ſame, *viz.* 112080 pence; as by the operation appeareth.

467 pounds
240
1868
934
Facit 112080 pence

Queſt. 3. In 5673 l. how many farthings? Firſt multiply the given number by 20, to bring it into ſhillings, and it produceth 113460 ſhillings; then multiply that product by 12, to bring it into pence, and it produceth 1361520 pence; then, laſtly, multiply the pence by 4, and it produceth 5446080 farthings. See the operation.

5673 pounds
20
113460 ſhillings
12
22692
11346
1361520 pence
4
Facit 5446080 farthings

Or this queſtion might have been thus reſolved, *viz.* Multiply 5673 (the given number of pounds) by 960 (the number of farthings in a pound), and it produceth the ſame effect; as you may ſee by the work.

	5673	pounds	20	ſhillings
	960		12	
	34038		240	pence
	51057		4	
Facit	5446080	farthings	960	farthings

Otherwiſe thus: Firſt, bring the given number 5673 l. into ſhillings, and multiply the ſhillings by 48, the number of farthings in a ſhilling, and the ſame effect is thereby likewiſe produced, *viz.*

	5673	pounds	12	pence
	20		4	
	113460	ſhillings	48	farthings
	48			
	90768			
	45384			
Facit	5446080	farthings		

Theſe various ways of operation are expreſſed to inform the judgment of the learner with the reaſon of the rule. More ways may be ſhewn, but theſe are ſufficient even for the meaneſt capacities.

Queſt. 4. In 458 l. 16 s. 7 d. 3 qrs. how many farthings? To reſolve this queſtion, conſider the ſeventh rule of this chapter, and work as you are there directed, and you will find the aforeſaid given numbers to amount to 440479 farthings, *viz.*

	l. *s.* *d.* *q.*	
	458—16—7—3	
	20	
	9160	
Add	16	ſhillings
Sum	9176	ſhillings
	12	
	18352	
	9176	
	110112	
Add	7	pence
Sum	110119	pence
	4	
	440476	
Add	3	farthings
Sum	440479	farthings

This laſt queſtion, or any other of this kind, *viz.* where the number given to be reduced conſiſteth of ſeveral denominations, may be more conciſely reſolved thus, *viz.* When you multiply the pounds by 20, to bring them into ſhillings, to the product of the firſt figure add the figure ſtanding in the place of units in the denomination of ſhillings: but becauſe the firſt figure in the multiplier is 0, I ſay, 0 times 8 is nothing, but 6 is 6, which I put down for the firſt figure in the product. Then, becauſe the multiplier is 0, I go on no further with it; for if I ſhould, the whole product would be 0; but proceed. And when I come

	l. *s.* *d.* *q.*	
	458—16—7—3	
	20	
	9176	ſhillings
	12	
	18359	
	9176	
	110119	pence
	4	
Facit	440479	farthings

to multiply by the ſecond figure in the multiplier, to the product of it I add the figure ſtanding in the place of tens in the denomination of ſhillings, which is 1; ſaying, 2 times 8 is 16 and (the ſaid figure) 1 is 17. Then I ſet down 7, and carry the unit to the product of the next figure, as is directed in the fifth rule of the ſixth chapter foregoing, and I finiſh the work. So that now you may have the whole product and ſum of ſhillings at one operation, which is the ſame as before. And when you multiply the ſhillings by 12, to bring them into pence, after the ſame manner, add to the product the number ſtanding in the denomination of pence; and ſo when you multiply the pence by 4, to bring them into farthings, add to the product the number ſtanding under the denomination of farthings. See the laſt queſtion thus wrought on the margin of the preceding page.

After the method laſt preſcribed (which, if rightly conſidered, differeth not any thing from the ſeventh rule of this chapter) are all the following examples that are of the ſame nature wrought and reſolved.

Queſt. 5. In 4375866 farthings, I demand how many pounds, ſhillings, pence, and farthings?

To reſolve this queſtion; firſt, I divide the given number of farthings by 4, and the quotient is 1093966 pence; and there remaineth 2 after the diviſion is ended; which by the eighth rule foregoing, is 2 farthings. Then I divide 1093966 pence by 12, and the quotient is 91163 ſhillings; and there remaineth 10 after diviſion; which, by the ſaid eighth rule, is ſo many pence, *viz.* 10 d. Then I divide 91163 ſhillings by 20, and the quotient is 4558 l. and there remaineth 3 ſhillings. So the work is finiſhed, and I find that in 4375866 farthings, there are 4558 l. 3 s. 10 d. 2 qrs. See the operation following.

```
              12)          2|0       l.
4) 4375866 (1093966   (9116|3  (4558
   .......   .....      ....
   4          108        8
   ---        ---        ---
   37          13        11
   36          12        10
   ---        ---        ---
    15         19        11
    12         12        10
    ---        ---       ---
     38        76        16
     36        72        16
     ---       ---       ---
      26        46       (3) s.
      24        36
      ---       ---
       26      (10) d.
       24
       ---
       (2) qrs.
```

l. s. d. qrs.

Facit 4558——3——10——2

Queſt. 6. In 4386 l. I demand how many groats?

To reſolve this queſtion, I reduce the given number of pounds into ſhillings, and they are 87720 ſhillings. Now I conſider, that in a ſhilling are 3 groats, therefore I multiply the ſhillings by 3, and it produceth 263160 groats. See the work.

```
       4386 pounds
         20
       ------
       87720 ſhillings
           3
       ------
Facit 263160 groats
```

This queſtion might have been otherwiſe reſolved thus, *viz.* Conſidering that in a pound (or 20 ſhillings) there are 3 times 20 groats, which makes 60; therefore I multiply the number of pounds given by 60, and it produceth the ſame effect at one operation. See the margin.

```
       4386 pounds
         60 groats in 20 s.
       ------
Facit 263160 groats in 4386 l.
```

Queſt. 7. In 43758 threepences, I deſire to know how many pounds?

To reſolve this, and many ſuch like queſtions; firſt, I divide my given number of threepences by 4, becauſe 4 threepences are a ſhilling, and the quotient is 10939 ſhillings; and there remaineth 2 after the diviſion is ended, which is 2 threepences, (by the eighth rule of this chapter), which are equal in value to 6 d. Then I divide 10939 ſhillings by 20, and the quotient giveth 546 l. and 19 s. remains: ſo that I conclude, in 43758 pieces of three pence *per* piece, there are 546 l. 19 s. 6 d. as by the work appeareth.

```
                2|0          l.   s.   d.
4) 43758  (1093|9   (546—19—6
   ...      ...

   4         10
   ───       ───
   37         9
   36         8
   ───       ───
    15        13
    12        12
    ───       ───
     38       19 ſhillings
     36
     ───
    (2) threepences, or 6 d.
```

This queſtion might have been otherwiſe reſolved thus, *viz.* Firſt, multiply the given number of threepences (43758) by 3, the number of pence in threepence, and the product (*viz.* 131274) is the number of pence equal to the given number of threepences; which number of pence may be brought into pounds, by dividing by 12 and by 20, and the quotient you will find to be equal to the former work, *viz.* 546 l. 19 s. 6 d.

```
      43758
          3
     ———————   2|0        l.   s.   d.
12) 131274   (1093|9   (546—19—6
    ......    ...
    12        10
    ———       ———
    112        9
    108        8
    ———       ———
     47       13
     36       12
    ———       ———
    114 Rem. (19) ſhillings
    108
    ———
Rem. (6) pence
```

Or thus: Divide the given number of threepences by the number of threepences in a pound or 20 ſhillings, (which you will find to be 80, if you multiply 20 s. by 4, the number of threepences in a ſhilling), and you will find the quote to be 546 l. as before, and a remainder of 78 threepences; and if you divide thoſe 78 threepences by 4, (becauſe there are 4 threepences in a ſhilling), you will find the quote to be 19 s. and 2 threepences remain, which are equal to 6 d. which is the ſame that was before found.

```
8|0) 4375|8 (546—19—6        20
     ...                       4
    40                        ——
    ———                       80
    37
    32
    ———
     55
     48
    ————
  4) 78 (19 s.
     ..
     4
     —
     38
     36
     —
     (2) threepences, or 6 d.
```

Queſt. 8. In 4785 l. 13 s. how many pieces of 13½ d. *per* piece?

This queſtion cannot be reſolved by reduction deſcending or aſcending abſolutely, (becauſe 13½ d. is no even part of a pound), but rather by them both jointly, *viz.* by multiplication and diviſion. For if you bring the number given into halfpence, and divide the halfpence by the halfpence in 13½, *viz.* 27, the quotient will be the anſwer: So having brought 4785 l. 13 s. into halfpence, I find it makes 2297112; which I divide by 27, (becauſe there are ſo many halfpence in 13½ d.), and the quote gives 85078 pieces of 13½ d. and 6 halfpence remain over and above. Obſerve the work following.

l. *s.*	*s.*
4785—13	13½ d.
20	2
95713 ſhillings	27 halfpence
24 halfpence in a ſhilling	
382852	
191426	
2297112 halfpence in the given number	

27) 2297112 (85078 pieces of 13½ d.

216

137
135

211
189

222
216

Rem. (6) halfpence

It would have produced the ſame anſwer, if you had reduced your given number into farthings, and divided by the farthings in 13½ d. *viz.* 54; (for always the dividend and the diviſor muſt be of one denomination); and then you would have had a remainder of 12 farthings, which are equal in value to the former remainder of 6 halfpence; as you may prove at your leiſure.

Queſt. 9. In 540 dollars, at 4 s. 4 d. *per* dollar, how many pounds Sterling?

Firſt, bring your given number of dollars into pence, and then your pence into pounds, according to the former directions. Thus, in 4 s. 4 d. (*viz.* a dollar) you will find 52 pence; by which multiply 540 dollars, and it produceth 28080 pence; which if you divide by 240, (the pence in one pound), the quotient will give you 117 l. which are equal in value to 540 dollars, at 4 s. 4 d. *per* dollar.

```
                    s.  d.
        540         4 — 4
         52            12
        ----           --
        108            52
       270
24|0)  2808|0 (117 l.
        ...
        24
        --
         40
         24
         --
         168
         168
         ---
         (0)
```

The foregoing queſtion might have been otherwiſe wrought thus; *viz.* Multiply (540) your given number of dollars, by 13, the number of groats in a dollar (or 4 s. 4 d.), and it produceth 7020 groats; which divide by 60 (the groats in 1 pound or 20 ſhillings), and the quote is 117 l. as before. See the work.

```
                  s.  d.
       540        4 — 4
        13          3
       ---         --
       162         13
       54
6|0)  702|0 (117 l.
       ...
       6
       -
       10
       6

       42
       42
       --
       (0)
```

Queſt. 10. In 547386 pieces of 4½ d. *per* piece, I demand how many pounds, ſhillings, and pence?

Firſt, bring your given number of four pence halfpenny all into halfpence, which you will do if you multiply by 9, the number of halfpence in 4½ d. and the product is 4926474 halfpence; which are brought into pounds, if you divide them by 24 (the halfpence in a ſhilling), and 20 (the ſhillings in a pound). It makes 10263 l. 9 s. 9 d. as by the work.

d.
4½
2
—
9 halfpence

547386
9
———
24) 4926474 (20526|9 2|0) (10263 *l.*
48 2
126 5
120 4
64 12
48 12
167 6
144 6
234 Rem. (9) ſhillings
216

l. *s.* *d.*
Facit 10263—9—9

Rem. (18) halfpence, or 9 d.

Queſt. 11. In 4386 l. I demand how many pieces of 6 d. of 4 d. and of 2 d. of each an equal number? that is to ſay, What number of ſixpences, groats, and twopences, will make up 4386 l. and the number of each equal?

The way to reſolve queſtions of this nature, is, to add the ſeveral pieces into which the given number is to be brought, into one ſum, and to reduce the given number into the ſame denomination with their ſum, and to divide the ſaid given number, ſo reduced, by the ſaid ſum, and the quotient will give you the exact number of each piece. And after the ſame method will we proceed to reſolve the preſent queſtion, *viz.*

4386 pounds
240 pence
———
17544
8772
———
12) 1052640 (87720
. . . . .
96
———
92
84
———
86
84
———
24
24
———
(0)

d.
6
4
2
——
Sum 12 pence

d. d. d.
Facit 87720 pieces of 6—4—2

So that I conclude by the operation, that 87720 ſixpences, 87720 groats, and 87720 twopences, are juſt as much as (or equal to) 4386 l. : or if you admit of 5 s. to be thus divided, it is equal to 5 ſixpences, and 5 fourpences or groats, and 5 twopences. “ For if two right “ lines (or two numbers) be given, and one of them be divided into as many parts or ſegments as you pleaſe, the “ rectangle (or product) comprehended under the two “ whole right lines (or numbers given) ſhall be equal to “ all the rectangles (or products) contained under the “ whole line (or number), and the ſeveral ſegments (or “ parts) into which the other line (or number) is divided. “ *Eucl.* 2. 1.”

Another queſtion of the ſame nature with the laſt, may be this following, *viz.*

Queſt. 12. A merchant is deſirous to change 148 l. into pieces of 13½ d. of 12 d. of 9 d. of 6 d. and of 4 d. and he will have of each ſort an equal number of pieces; I deſire to know the number?

Do as you were taught in the laſt queſtion, *viz.* Add the ſeveral pieces together, and reduce the ſum into halfpence; then reduce the ſum to be changed, *viz.* 148 l. into the ſame denomination, and divide the greater by the leſſer, and in the quotient you will find the anſwer, *viz.* 798 is the number of each of the pieces required, and 18 remaineth, which is 18 halfpence, by the eighth rule of this chapter. See the work as followeth.

l.	*d.*
148	13½
240 pence in a pound	12
	9
592	6
296	4
35520 pence in 148 l.	Sum 44½
2	2
71040 halfpence	89 halfpence

89) 71040 (798 pieces of each ſort
623
874
801
730
712
Rem. (18) halfpence

The truth of the two foregoing operations will thus be proved, *viz.* Multiply the anſwer by the parts or pieces into which the given number was reduced; and having added the ſeveral products together, if their ſum be equal to the given number, the anſwer is right; otherwiſe not.

So the anſwer to the 11th queſtion was 87720; which is proved as followeth, *viz.*

87720

		l.
87720	ſixpences make ———	2193
	fourpences make ——	1462
	twopences make ——	731
	The total ſum of them	4386

which was the ſum given to be changed.

The anſwer to the 12th queſtion was 798, and 18 halfpence remained after the work was ended. Now, the truth of the work may be proved as the former was, *viz.*

		d.		*l.*	*s.*	*d.*
798	pieces of	$13\frac{1}{2}$	make	44	17	9
	pieces of	12	make	39	18	0
	pieces of	9	make	29	18	6
	pieces of	6	make	19	19	0
	pieces of	4	make	13	6	0
and 18 halfpence, or 9 d. remains				0	0	9
The total ſum of them				148	0	0

which total ſum is equal to the number that was firſt given to be changed; and therefore the operation was rightly performed.

Reduction of Troy weight.

We now come to give the learner ſome examples in Troy weight; wherein we ſhall be brief, having given ſo large a taſte of reduction in the former examples of coin. And now the learner muſt be mindful of the table of Troy weight delivered in the ſecond chapter of this book.

Queſt. 13. In 482 lb. 7 oz. 13 pw. 21 grs. how many grains?

Multiply by 12, by 20, and by 24, taking in the figures ſtanding in the ſeveral denominations, according to the directions given in the ſeventh rule of this chapter, and you will find the product to be 2780013 grains, which is the number required, or anſwer to the queſtion. See the whole work as followeth.

```
      lb.    oz.   pw.    gr.
      482 —— 7 —— 13 —— 21
       12
     ————
      971
     482
     —————
     5791 ounces
       20
     —————
   115833 penny-weight
       24
    ——————
   463333
  231668
  ———————
```

Facit 2780013 grains.

Queſt. 14. In 2780013 grains, I demand how many pounds, ounces, penny-weights, and grains?

This is but the foregoing queſtion inverted, and is reſolved by dividing by 24, by 20, and by 12; and the anſwer is 482 lb. 7 oz. 13 pw. 21 gr.

```
                      2|0           12)           l.
 24) 2780013        (11583|3       (5791         (482
     ·······         ····· ·        ···

     24              10             48
     ——              ——             ——
     38              15             99
     24              14             96
     ———             ——             ——
     140             18             31
     120             18             24
     ———             ——             ——
      200             3   Rem.     (7) ounces
      192             2
      ———            ——
       81     Rem.  (13) penny-weight
       72
       ——                           lb. oz. pw. gr
       93                    Facit 482—7—13—21
       72
       ——
Remains (21) grains
```

Queſt. 15. A merchant ſent to a goldſmith 16 ingots of ſilver, each containing in weight 2 lb. 4 oz. and ordered it to be made into bowls of 2 lb. 8 oz. *per* bowl, and tankards of 1 lb. 6 oz. *per* piece, and ſalts of 10 oz. 10 pw. *per* ſalt, and ſpoons of 1 oz. 18 pw. *per* ſpoon, and of each an equal number; I deſire to know how many of each ſort he muſt make?

This queſtion is of the ſame nature with the 11th and 12th queſtions foregoing, and may be anſwered after the ſame method, *viz.* Firſt, add the weight of the ſeveral veſſels into which the ſilver is to be made, into one ſum, and reduce them to one denomination, and they make 1248 penny weights; then reduce the weight of the ingot into the ſame denomination, *viz.* penny-weights, and it makes 560 penny-weights; and multiply them by the number of ingots, *viz.* 16, and the product will give you the weight of the 16 ingots, *viz.* 8960 penny-weights; then divide the product by the weight of the veſſels, *viz.* 1248, and the quotient giveth you the anſwer to the queſtion, *viz.* 7, and 224 pw. remaineth over and above.

lb. *oz.*
2 —— 4
12
——
28
20
——
560 penny-weights
16 ingots
——
336
56
——
1248) 8960 (7 veſſels of each ſort
8736
——
Rem. (224) penny-weights

lb. *oz.* *pw.*
2 —— 8 —— 0
1 —— 6 —— 0
0 —— 10 —— 10
0 —— 1 —— 18
——
Sum 5 —— 2 —— 8
12
——
62 oz.
20
——
1248 pw.

The proof of the work is as followeth, viz.

		lb.	oz.	pw.		lb.	oz.	pw.
7	bowls of	2	8	0	*per* bowl is	18	8	0
	tank. of	1	6	0	*per* tank. is	10	6	0
	ſalts of	0	10	10	*per* ſalt is	6	1	10
	ſpoons of	0	1	18	*per* ſpoon is	1	1	6
	224 penny-weight remaining					0	11	4
	Total ſum					37	4	0

So that you ſee the ſum of the weight of all the veſſels, together with the remainder, is 37 lb. 4 oz. which is equal to the weight of the 16 ingots delivered. For if 37 lb. 4 oz. be reduced to penny-weights, it makes 8960.

Reduction of Avoirdupois weight.

In reducing Avoirdupois weight, the learner muſt have recourſe to the table of Avoirdupois weight delivered in the ſecond chapter foregoing.

Queſt. 16. In 47 C. 1 qr. 20 lb. how many ounces? Multiply by 4, by 28, and by 16; and the laſt product will be the anſwer, *viz.* 84992 ounces.

```
 C.  qr. lb.
 47—1—20
  4
 ————
 189 quarters
  28
 ————
 1512
 380
 ————
 5312 lb.
   16
 ————
 31872
 5312
 ————
*Facit*  84992 ounces
```

Queſt. 17. In 84992 ounces, I demand how many C. qrs. lb. and oz.?

This is the foregoing queſtion inverted; and will be reſolved if you divide by 16, by 28, and by 4; and the anſwer is 47 C. 1 qr. 20 lb. equal to the given number in the foregoing queſtion.

28) 4) *C. qr. lb. oz.*
16) 84992 (5312 (189 (47—1—20—0
....
80 28 16
49 251 29
48 224 28
19 272 (1) qr.
16 252
32 (20) pounds
32
(0)

Queſt. 18. A chapman buyeth of a grocer 4 C. 1 qr 14 lb. of pepper, and ordereth it to be made up into parcels of 14 lb. of 12 lb. of 8 lb. of 6 lb. and of 2 lb. and of each parcel an equal number; now I would know the number of each parcel?

This example is of the ſame nature with the 11th, 12th, and 15th queſtions foregoing; and after the ſame manner is reſolved. See the operation as followeth.

C. qrs. lb. *lb.*
4——1——14 14
4 12
8
17 6
28 2
140 42 lb.
35
42) 490 (11
42
70 *Facit* 11 parcels of each
42
Rem. (28) pounds

Reduction of liquid meaſure.

Queſt. 19. In 45 tun of wine, how many gallons? Multiply by 4, and by 63; the product is 11340 gallons for the anſwer.

45
4
180
63
54
108
Facit 11340 gallons

Queſt. 20. In 34 rundlets of wine, each containing 18 gallons, I demand how many hogſheads?

Firſt, find how many gallons are in the 34 rundlets, which you may do if you multiply 34 by 18, the content of a rundlet, and the product is 612 gallons; which you may reduce into hogſheads, if you divide them by 63, and the quote will be 9 hogſheads and 45 gallons. See the work.

34
18
272
34
63) 612 (9 hhds
567
Remains (45) gallons
Facit 9 hhds 45 gall.

Queſt. 21. In 12 tuns, how many rundlets of 14 gallons *per* rundlet?

Reduce your tuns into gallons, and divide them by 14, the gallons in a rundlet, and the quotient (216) is your anſwer. See the work in the margin.

12
4
48
63
144
288
14) 3024 (216 rundlets
28
22
14
84
84 *Facit* 216 run.

Reduction of long measure.

Queſt. 22. I demand how many furlongs, poles, inches, and barley-corns will reach from London to York, it being accounted 151 miles?

151 miles
8 furlongs in a mile

1208 furlongs
40 poles in a furlong

48320 poles
11 half-yards in a pole

4832
4832

531520 half-yards
18 inches in half a yard

425216
53152

9567360 inches
3 barley-corns in an inch

Facit 28702080 barley corns in 151 miles.

Queſt. 23. The circumference of the earth (as all other circles are) is divided into 360 degrees, and each degree into 60 minutes, which, upon the ſuperficies of the earth, are equal to 60 miles; now, I demand how many miles, furlongs, perches, yards, feet, and barley-corns, will reach round the globe of the earth?

	360	degrees
	60	minutes or miles in a degree
	21600	miles about the earth
	8	furlongs in a mile
	172800	furlongs about the earth
	40	perches in a furlong
	6912000	poles or perches about the earth
	11	half-yards in a perch
	6912	
	6912	
	2)76032000	half-yards about the earth
	(38016000	yards, viz. the half yards divided by 2
	3	
	114048000	feet about the earth
	12	inches in a foot
	228096	
	114048	
	1368576000	inches about the earth
	3	barley-corns in an inch
Facit	4105728000	barley-corns

And ſo many will reach round the world, the whole being 21600 miles. So that if any perſon were to go round, and go 15 miles every day, he would go the whole circumference in 1440 days, which is 3 years, 11 months, and 15 days.

Reduction of time.

Queſt. 24. In 28 years, 24 weeks, 4 days, 16 hours, 30 minutes, how many minutes?

Years. weeks. days. hours. minutes.
28——24——4——16——30
52 weeks in a year
60
142
1480 weeks
7
10364 days
24
41462
20729
248752 hours
60
14925150 minutes

Note, That in reſolving the laſt queſtion after the method expreſſed, there are loſt in every year 30 hours; for the year conſiſteth of 365 days and 6 hours; but by multiplying the years by 52 weeks, which is 364 days, you loſe 1 day and 6 hours every year. Wherefore, to find an exact anſwer, bring the odd weeks, days, and hours, into hours, and then multiply the years by the number of hours in a year, *viz.* 8766, and to the product add the hours contained in the odd time, and you have the exact time in hours; which bring into minutes, as before. See the laſt queſtion thus reſolved.

```
                *Days. hours.*          *Weeks. days. hours.*
      28          365—6                   24—4—16
    8766           24                      7
    ----          ----                    ---
     172          1466                    172
     172          730                      24
     197          ----                    ---
    228           8766 hours in a year    694
  ------                                 345
  249592  hours                          ----
      60                                 4144 hours
  --------
  14975520   min. in 28 years and 4144 hours
```

So you ſee that according to the method firſt uſed to reſolve this queſtion, the hours contained in the given time, are 248752; but according to the laſt, beſt, or trueſt method, they are 249592, which exceeds the former by 840 hours.

But for moſt occaſions it will be ſufficient to multiply the given years by 365; and to the product add the days in the odd time, if there be any, and then there will be only a loſs of 6 hours in every year; which may be ſupplied by taking a fourth part of the given years, and adding it to the contained days, and you have your deſire.

Queſt 25. In 438657540 minutes, how many years? *Facit* 834 years, 4 days, 19 hours.

```
                          8766          *Years. days. hours.*
  6|0) 43865754|0    (7310959         (834———4———19
       · · · · · · ·       · · ·

        42              70128
       ----            -------
         18              29815
         18              26298
        ----            -------
           6               35179
           6               35064
          ----            -------
           57         24)  115  (4 days
           54                ·
          ----
            35               96
            30       Rem.   ----
           ----             (19) hours
             54
             54
            ----
            (0)
```

Queſt. 26. I deſire to know how many hours and minutes it is ſince the birth of our Saviour Jeſus Chriſt to this preſent year, being accounted 1677 years?

```
      1677
      8766 hours in a year
     -----
     10062
    10062
   11739
  13416
  --------
  14700582 hours in 1677 years
        60
  --------
 882034920 minutes in 1677 years.
```

This queſtion is of the ſame nature with the 24th foregoing, and after the ſame manner is reſolved, *viz.* Multiply the given number of years by 8766, and the product is 14700582 hours; and that by 60, and the product is 882034920 minutes. See the work.

Note, That as multiplication and diviſion do interchangeably prove each other; ſo reduction deſcending and aſcending, prove each other by inverting the queſtion, as the 13th and 14; and likewiſe the 16th and 17th queſtions foregoing, by inverſion, do interchangeably prove each other. The like may be performed for the proof of any queſtion in reduction whatſoever.

Thus far have we diſcourſed concerning ſingle arithmetic, whoſe nature and parts are defined in the ſecond, eighth, ninth, and tenth definitions of the third chapter of this book. For although reduction is not reckoned or defined among the parts of ſingle arithmetic; yet, conſidered abſtractedly, it is the proper effect of multiplication and diviſion. And as for the extraction of roots, (which ought to be handled in the next place as parts of ſingle arithmetic), we ſhall omit it in this place; and refer the learner to Mr Cocker's *decimal arithmetic,* which is (with great care and pains) now publiſhed together with his *logarithmetical arithmetic,* ſhewing the geneſis or fabric of the logarithms, and their general uſe in arithmetic, *&c.*; as alſo his *algebraical arithmetic,* containing the doctrine of compoſing and reſolving an equation, with all other rules neceſſary for the underſtanding of that myſterious art, *&c.*

CHAP.

CHAP. IX.

Of comparative arithmetic, viz. The relation of numbers one to another.

1. COmparative arithmetic, is that which is wrought by numbers, as they are conſidered to have relation one to another; and this conſiſts either in quantity or in quality. *Vid. Boetius's arith. lib.* 1. *cap.* 21.

2. Relation of numbers in quantity, is the reference or reſpect that the numbers themſelves have one to another; where the terms or numbers propounded are always two, the firſt called *the antecedent*, and the other *the conſequent*. *Vid. Wing. arith. cap.* 34.

3. The relation of numbers in quantity conſiſts in the differences, or in the rate or reaſon that is found betwixt the terms propounded; the differences of two numbers being the remainder found by ſubtraction; but the rate or reaſon betwixt two numbers is the quotient of the antecedent divided by the conſequent. So 21 and 7 being given, the difference betwixt them will be found to be 14; but the rate or reaſon that is betwixt 21 and 7, will be found to be triple reaſon; for 21 divided by 7, quotes 3, the reaſon or rate. *Vid. Alſted. mathemat. lib.* 2. *cap.* 11. *&* 12.

4. The relation of numbers in quality, (otherwiſe called *proportion*), is the reference or reſpect that the reaſon of numbers have one to another: therefore the terms given ought to be more than two. Now, this proportion, or reaſon between numbers relating one to another, is either arithmetical, or geometrical. *Vid. Alſted. mathemat. lib.* 2. *cap.* 21.

5. Arithmetical proportion (by ſome called *progreſſion*) is, when divers numbers differ one from another by equal reaſon; that is, have equal differences.

So this rank of numbers, 3, 5, 7, 9, 11, 13, 15, 17, differ by equal reaſon, *viz.* by 2; as you may prove.

6. In a rank of numbers that differ by arithmetical proportion, the ſum of the firſt and laſt term being multiplied by half the number of terms, the product is the total ſum of all the terms.

Or, if you multiply the number of the terms, by the half ſum of the firſt and laſt terms, the product is the total ſum of all the terms.

So, in the former progreſſion given, 3 and 17 is 20, which multiplied by 4, *viz.* half the number of terms, the product gives 80, the ſum of all the terms: or multiply 8 (the number of terms) by 10 (half the ſum of the firſt and the laſt term), the product gives 80, as before.

So alſo 21, 18, 15, 12, 9, 6, 3, being given, the ſum of all the terms will be found to be 84; for here the number of terms is 7, and the ſum of the firſt and laſt (*viz.* 21 and 3) is 24, half whereof (*viz.* 12) multiplied by 7, produceth 84, the ſum of the terms ſought.

7. Three numbers that differ by arithmetical proportion, the double of the mean, or middle number, is equal to the ſum of the extremes.

So 9, 12, and 15 being given, the double of the mean 12 (*viz.* 24) is equal to the ſum of the two extremes 9 and 15.

8. Four numbers that differ by arithmetical proportion, either continued or interrupted, the ſum of the two means is equal to the ſum of the two extremes.

So 9, 12, 18, 21, being given, the ſum of 12 and 18 will be equal to the ſum of 9 and 21, *viz.* 30. Alſo 6, 8, 14, 16, being given, the ſum of 8 and 14 is equal to the ſum of 6 and 16, *viz.* 22, &c. *Vid. Wing. arith. cap.* 35.

9. Geometrical proportion (by ſome called *geometrical progreſſion*) is, when divers numbers differ according to right reaſon.

So 1, 2, 4, 8, 16, 32, 64, &c. differ by double reaſon; and 3, 9, 27, 81, 243, 729, differ by triple reaſon; 4, 16, 64, 256, &c. differ by quadruple reaſon, &c.

10. In any numbers that increaſe by geometrical proportion, if you multiply the laſt term by the quotient of any one of the terms divided by another of the terms, which being leſs, is next unto it; and having deducted or ſubtracted the firſt term out of that product, divide the remainder by a number that is an unit leſs than the ſaid quotient, the laſt quote will give the ſum of all the terms.

So 1, 2, 4, 8, 16, 32, 64, being given; firſt, I take one of the terms, *viz.* 8, and divide it by the term, which is leſs, and next to it, *viz* by 4, and the quotient is 2; by which I multiply the laſt term 64, and the product is 128; from whence I ſubtract the firſt term, *viz.* 1, the remainder is 127; which divided by the quotient 2, made leſs by 1, *viz.* 1, the quote is 127, for the ſum of all the given terms; as by the work in the margin.

```
     64
4) 8 (2
  ----
    128
      1
  ----
1) 127 (127
```

So if 4, 16, 64, 256, 1024, were given, the ſum of all the terms will be found to be 1364. For firſt, I divide 64, one of the terms, by the next leſſer term, and the quotient is 4; by which I multiply the laſt term, 1024, and it produceth 4096; from whence I ſubtract the firſt term, 4, and the remainder is 4092; which I divide by the quote, leſs by 1, *viz.* 3, and the quote is 1364, for the total ſum of all the terms; as *per* margin.

```
     1024
16) 64 (4
    ----
     4096
        4
    ----
3) 4092 (1364
```

So likewiſe if 2, 6, 18, 54, 162, 486, were given, the ſum or total of all the terms will be found to be 728. See the work.

```
    486
6) 18 (3
  -----
   1458
      2
  -----
2) 1456 (728
```

11. Three geometrical proportionals being given, the ſquare of the mean is equal to the rectangle or product of the extremes.

So 8, 16, 32, being given, the ſquare of the mean, *viz.* 16, is 256, which is equal to the product of the extremes 8 and 32; for 8 times 32 is equal to 256.

12. Of four geometrical proportional numbers given, the product of the two means, is equal to the product of the two extremes.

So 8, 16, 32, 64, being given, I ſay, that the product of the two means, *viz.* 16 times 32, which is 512, is equal to 8 times 64, the product of the extremes.

Alſo if 3, 9, 21, 63, were given, which are interrupted, I ſay, 9 times 21 is equal to 3 times 63, which is equal to 189.

From hence ariſeth that precious gem in arithmetic, which for the excellency thereof is called *the golden rule,* or *rule of three.*

C H A P. X.

The ſingle rule of Three direct.

1. THE rule of three (not undeſervedly called *the golden rule)* is that by which we find out a fourth number in proportion unto three given numbers, ſo as this fourth number ſought may bear the ſame rate, reaſon, or proportion, to the third (given) number, as the ſecond doth to the firſt; from whence it is called *the rule of proportion.*

2. Four numbers are ſaid to be proportional, when the firſt containeth, or is contained by the ſecond, as often as the third containeth, or is contained by the fourth. *Vid. Wing. arith. cap.* 8. *ſect.* 4.

So theſe numbers are ſaid to be proportionals, *viz.* 3, 6, 9, 18; for as often as the firſt number is contained in the ſecond, ſo often is the third contained in the fourth, *viz.* twice. Alſo 9, 3, 15, 5, are ſaid to be proportionals; for as often as the firſt number containeth the ſecond, ſo often the third number containeth the fourth, *viz.* 3 times.

3. The rule of three is either ſimple or compound.

4. The ſimple (or ſingle) rule of three, conſiſteth of 4 numbers; that is to ſay, it hath three numbers given to find out a fourth. And this is either direct or inverſe. *Vid. Alſted. math. lib.* 2. *cap.* 13.

5. The ſingle rule of three direct, is, when the proportion of the firſt term is to the ſecond, as the third is to the fourth; or when it is required that the number ſought, *viz.* the fourth number, muſt have the ſame proportion to the ſecond, as the third hath to the firſt.

6. In the rule of three, the greateſt difficulty is (after the queſtion is propounded) to diſcover the order of the three terms, *viz.* which is the firſt, which is the ſecond, and which the third. Which that you may underſtand, obſerve, That, of the three given numbers, two always are of one kind, and the other is of the ſame kind with the proportional number that is ſought; as in this queſtion, *viz.* If 4 yards of cloth coſt 12 ſhillings, what will 6 yards coſt at that rate? Here the two numbers of one kind are 4 and 6, *viz.* they both ſignify ſo many yards; and 12 ſhillings is the ſame kind with the number ſought, for the price of ſix yards is ſought.

Again obſerve, that of the three given numbers, thoſe two that are of the ſame kind, one of them muſt be the firſt, and the other the third; and that which is of the ſame kind with the number ſought, muſt be the ſecond number in the rule of three. And that you may know which of the ſaid numbers to make your firſt, and which your third, know this, that to one of theſe two numbers there is always affixed a demand, and that number upon which the demand lieth, muſt always be reckoned the third number: As in the forementioned queſtion, the demand is affixed to the number 6; for it is demanded, what 6 yards will coſt; and therefore 6 muſt be the third number, and 4, which is of the ſame denomination or kind with it, muſt be the firſt: and conſequently the number 12 muſt be the ſecond. And then the numbers being placed in the forementioned order, will ſtand as followeth, *viz.*

Yards.		*s.*		*Yards.*
4	:	12	: :	6

7. In the rule of three direct, (having placed the number as is before directed), the next thing to be done will be, to find out the fourth number in proportion: which that you may do, multiply the ſecond number by the third, and divide the product thereof by the firſt; or (which is all one) multiply the third term (or number) by the ſecond, and divide the product thereof by the firſt; and the quotient thence ariſing is the fourth number in a direct proportion; and is the number ſought, or anſwer to the queſtion; and is of the ſame denomi-

nation that the ſecond number is of. As thus: Let the ſame queſtion be again repeated, *viz.* If 4 yards of cloth coſt 12 ſhillings, what will 6 yards coſt?

Having placed the numbers according to the ſixth rule of this chapter, I multiply (the ſecond number) 12 by (the third number) 6, and the product is 72; which product I divide by (the firſt number) 4, and the quotient thence ariſing is 18: which is the fourth proportional, or number ſought, *viz.* 18 ſhillings, (becauſe the ſecond number is ſhillings); which is the price of 6 yards, as was required by the queſtion. See the work on the margin.

```
   Yards.     s.    Yards.     s.
   If 4   :   12  ::  6   :   18
               6
              —
          4)  72 (18 ſhillings
              ..
               4
              —
              32
              32
              —
              (0)
```

Queſt. 2. If 7 C. of pepper coſt 21 l. how much will 16 C. coſt at that rate?

To reſolve this queſtion, I conſider, that (according to the ſixth rule of this chapter) the terms or numbers ought to be placed thus, *viz.* the demand lying upon 16 C. it muſt be the third number, and that of the ſame kind with it muſt be the firſt, *viz.* 7 C. and 21 l. (being of the ſame kind with the number ſought) muſt be the ſecond number in this queſtion. Then I proceed according to the ſeventh rule, and multiply the ſecond number by the third, *viz.* 21 by 16, and the product is 336; which I divide by the firſt number 7, and the quotient is 48 l. which is the value of 16 C. of pepper at the rate of 21 l. for 7 C. See the work on the margin.

```
   C.      l.      C.
   7   :   21  ::  16
           16
          ———
          126
          21
          ———
     7)   336 (48 l.
           ..
           28
          ———
            56
            56
            —
           (0)
```

8. If when you have divided the product of the ſecond and third numbers by the firſt, any thing remain

after diviſion is ended, ſuch remainder may be multiplied by the parts of the next inferior denomination that are equal to an unit (or integer) of the ſecond number in the queſtion; and the product thereof being divided by the firſt number in the queſtion, the quotient is of the ſame denomination with the parts by which you multiplied the remainder, and is part of the fourth number which is ſought. And furthermore, if any thing remain after this laſt diviſion is ended, multiply it by the parts of the next inferior denomination, equal to an unit of the laſt quotient, and divide the product by the ſame diviſor, *viz.* the firſt number in the queſtion, and the quote is ſtill of the ſame denomination with your multiplier. Follow this method, until you have reduced your remainder into the loweſt denomination, *&c.* An example or two will make this rule very plain, which may be theſe following.

Queſt. 3. If 13 yards of velvet (or any other thing) coſt 21 l. what will 27 yards of the ſame coſt at that rate?

Having ordered and wrought my numbers according to the ſixth and ſeventh rules of this chapter, I find the quotient to be 43 l. and there is a remainder of 8; ſo that I conclude the price of 27 yards to be more than 43 l.: and to the intent that I may know how much more, I work according to the foregoing rule, *viz.* I multiply the ſaid remainder 8 by 20, (becauſe the ſecond number in the queſtion was pounds), and the product is 160; which divided by the firſt number, *viz.* 13, it quotes 12, which are 12 ſhillings; and there is yet a remainder of 4: which I multiply by 12, (becauſe the laſt quotient was ſhillings), and the product is 48; which I divide by 13 (the firſt number), and the quotient is 3 d.; and yet there remaineth 9: which I multiply by 4, and the product is 36; which divided again by 13, it quotes 2 farthings; and there is yet a remainder of 10: which (becauſe it cometh not to the value of a farthing) may be neglected: or rather ſet after the 2 farthings over the diviſor, with a line between them; and then (by the 21ſt and 22d definitions of the firſt chapter of this book) it will be $\frac{10}{13}$ of a farthing. So that I conclude, that if 13 yards of velvet coſt 21 l. 27 yards of the ſame will coſt 43 l. 12 s. 3 d. $2\frac{10}{13}$ qrs.

which fraction is ten thirteenths of a farthing. See the operation as followeth.

Yards. *l.* *Yards.*
If 13 : 21 : : 27
27
147
42
13) 567 (43 l.
52
47
39
Remains 8
Multiply 20
13) 160 (12 s.
13
30
26
Remains 4
Multiply 12
13) 48 (3 d.
39
Remains 9
Multiply 4
13) 36 ($2\frac{10}{13}$ qrs.
26
Remains 10

l. *s.* *d.* *qrs.*
Facit 43—12—3—$2\frac{10}{13}$

Queſt. 4. Another example may be this following, *viz.* If 14 lb. of tobacco coſt 27 s. what will 478 lb. coſt at that rate ?

Work according to the laſt rule, and you will find it to amount to 921 s. 10 d. $1\frac{2}{14}$ qrs.; and, by the fifth rule of the 8th chapter, 921 s. may be reduced to 46 l. 1 s.: ſo that then the whole worth or value of the 478 lb. will be 46 l. 1 s. 10 d. $1\frac{2}{14}$ qrs. The whole work followeth.

lb. *s.* *lb.*
If 14 : 27 :: 478
27
3346
956
2|0
14) 12906 (92|1 (46 l.
126 8
30 12
28 12
26 1 s.
14
Remains 12
Multiply 12
24
12
14) 144 (10 d.
14
mains 4
Multiply 4
14) 16 ($1\frac{2}{14}$ qr.
14
Remains 2 *l.* *s.* *d.* *qrs.*
Facit 46—1—10—$1\frac{2}{14}$

9. In the rule of three it many times happeneth, that although the firſt and third numbers be homogeneal, that is of one kind, as both money, weight, meaſure, *&c.* yet they may not be of one denomination, or perhaps they may both conſiſt of many denominations: in which caſe you are to reduce both numbers to one denomination; and likewiſe your ſecond number (if it conſiſteth at any time of divers denominations) muſt be reduced to the leaſt name mentioned, or lower if you pleaſe: which being done, multiply the ſecond and third together, and divide by the firſt, as is directed in the ſeventh rule of this chapter.

And note, That always the anſwer to the queſtion is in the ſame denomination that your ſecond number is of, or is reduced to, as was hinted before.

Queſt. 5. If 15 ounces of ſilver be worth 3 l. 15 s. what are 86 ounces worth at that rate?

In this queſtion, the numbers being ordered according to the ſixth rule of this chapter, the firſt and third numbers are ounces, and the ſecond number is of divers denominations, *viz.* 3 l. 15 s. which muſt be reduced to ſhillings, and the ſhillings multiplied by the third number, and the product divided by the firſt, gives you the anſwer in ſhillings, *viz.* 430 ſhillings, which are reduced to 21 l. 10 s.

```
      oz.       l.     s.       oz.
   If 15   :    3——15    : :   86
                20
                ——
                75
                86
              ————
               450
              600
              ———— (2|0    l.   s.
       15) 6450 (43|0 (21——10
           ...     ..
            60      4
          ————     ——
            45      3
            45      2
           ———     ——
           (0)     10 ſhillings
```

In reſolving the laſt queſtion, the work would have been the ſame, if you had reduced your ſecond number into pence; for then the anſwer would have been 5160 pence, equal to 21 l. 10 s.; or if you had reduced the ſecond number into farthings, the quotient, or anſwer, would have been 20640 farthings, equal to the ſame; as you may prove at your leiſure.

Queſt. 6. If 8 lb. of pepper coſt 4 s. 8 d. what will 7 C. 3 qrs. 14 lb. coſt?

In this queſtion the firſt number is 8 lb. and the third is 7 C. 3 qrs. 14 lb. which muſt be reduced to the ſame denomination with the firſt, *viz.* into pounds, and the ſecond number muſt be reduced into pence; then multiply and divide according to the ſeventh rule foregoing, and you will find the anſwer to be 6174 pence, which is reduced into 25 l. 14 s. 6 d.

```
   lb.   s.  d.        C. qrs. lb.
If 8  :  4--8   : :   7—3—14
         12            4
         —             —
         56           31
                      28
                     ————
                     252
                      63
                     ————
                     882
                      56 ſecond number
                    —————
                    5292
                   4410
                   ——————      12)      2|0)     l.   s.   d.
                8) 49392     (6174    (51|4    (25 —14 — 6
                   ...        ....     ..
                   48         60       4
                   ————       ————     ——
                     13        17      11
                      8        12      10
                     ——        ——      ——
                     59        54      14 s.
                     56        48
                     ——        ————
                      32        6 d.
                      32
                      ——                 l.   s.   d.
                     (0)     Facit 25—14—6
```

Queſt. 7. If 3 C. 1 qr. 14 lb. of raiſins coſt 9 l. 9 s. what will 6 C. 3 qrs. 20 lb. of the ſame coſt?

Here the firſt and third numbers, each conſiſt of divers denominations, but muſt be brought both into one denomination, *&c.* as you ſee in the operation that followeth. The anſwer is 388 s. which is reduced into 19 l. 8 s.

```
  C. qr. lb.         l.  s.          C. qrs. lb.
If 3—1—14     :     9—9    : :     6—3—20
   4                20               4
  ——               ————             ——
  13               189               27
  28                                 28
 ————                               ————
 108                                216
 27                                  56
 ————                               ————
 378 pounds                          776 pounds
                                     189 ſecond number
                                    ————
                                    6984
                                   6208
                                   776
                                  ——————   2|0)     l.  s.
                           378)  146664  (38|8   (19—8
                                  • • •   • •
                                  1134     2
                                 ——————    —
                                  3326     18
                                  3024     18
                                 ——————   ————
                    l.  s.         3024      8 s.
            Facit 19—8           3024
                                  ————
                                   (0)
```

Queſt. 8. If in 4 weeks I ſpend 13 s. 4 d. how long will 53 l. 6 s. laſt me at that rate?

Anſ. 2238 days, equal to 6 years, 48 days. See the work in the following page.

```
     s.      d.      w.          l.     s.
If  13 ——— 4   :   4   : :   53 ——— 6
    12              7           20
    ——              ——        —————
    30          28 days       1066
    13                          12
    ———                       —————
    160                        2132
                              1066
                              —————
                             12792 pence
                                28 ſecond number
                             ——————
                            102336
                            25584
                            ——————      365)
                  16|0)   35817|6  (2238 (6 years
                           ....     2190
                           32       ————
                           ——    Rem. 48 days
                            38
                            32
                            ——
                            61         Years.     days.
                            48    Facit 6 ———— 48 96/160
                            ——
                           137
                           128
                           ———
                 Remains    96
```

Queſt. 9. Suppoſe the yearly rent of a houſe, a yearly penſion or wages be 73 l. I deſire to know how much it is *per* day?

Here you are to bring the year into days, and ſay, If 365 days require 73 l. what will 1 day require?

Now, when you come to multiply 73 by one, the product is the ſame; for 1 neither multiplieth nor divideth; and 73 cannot be divided by 365, becauſe the diviſor is bigger than the dividend; wherefore bring the 73 l. into ſhillings, and they make 1460; which divide by the firſt number 365, and the quote is 4 ſhillings for the anſwer: as you ſee in the work.

Days. *l.* *Day.*
If 365 : 73 : : 1
20

365) 1460 (4 s.
1460

(0)

Facit 4 s. *per* day

Queſt. 10. A merchant bought 14 pieces of broad cloth, each piece containing 28 yards, for which he gave after the rate of 13 s. 6½ d. *per* yard. Now, I deſire to know how much he gave for the 14 pieces at that rate?

Firſt, find out how many yards are in the 14 pieces; which you will do, if you multiply the 14 pieces by 28, (the number of yards in a piece), and it makes 392. Then ſay, If 1 yard coſt 13 s. 6½ d. what will 392 yards coſt? Work as followeth; and the anſwer you will find to be 127400 halfpence; which reduced, make 265 l. 8 s. 4 d.: for after you have multiplied your ſecond and third numbers together, the product is 127400; which, according to the ſeventh rule, ſhould be divided by the firſt number; but the firſt number is 1, which neither multiplieth nor divideth; and therefore the quotient or fourth number is the ſame with the product of the ſecond and third; which is in halfpence, becauſe the ſecond number was ſo reduced. See the work as followeth.

```
                           28
                           14
                          ----
                          112
                          28
                          ----
                          392 yards in the 14 pieces.
   Yard.     s.   d.      Yards.
   If 1  :  13—6½  ::     392
             12           325 the ſecond number
           ----          -----
             32          1960
            13           784
           ----         1176
            162         ------    2|0)       l.
              2     24) 127400   (530|8  (265
           ----          ....     ...
   Halfpence 325         120      4
                         ---      --
                          74      13
                          72      12
                         ---      --
                          200     10
                          192     10
                         ---      --
                                   8 ſhillings
       l.   s.  d.       Rem. 8 halfpence, or 4 d.
   Facit 265—8—4
```

Queſt. 11. A draper bought 420 yards of broad cloth, and gave for it after the rate of 14 s. 10¾ d. *per* ell Engliſh. Now, I demand how much he paid for the whole after that rate?

Bring your ells into quarters, and your given yards into quarters; the ell Engliſh is 5 quarters, and 420 yards are 1680 quarters: then ſay, If 5 quarters coſt 14 s. 10¾ d. (or 715 farthings), what will 1680 quarters coſt? *Facit* 250 l. 5 s. See the operation following.

```
       Ell.              Yards.
        1                 420
        5                   4
       ---               ----
        5                1680 qrs.

Qrs     s.   d.          Qrs.
If 5 :  14—10¾    : :    1680
        12                715
        --               ----
        28                840
       15                168
      ----              1176
      178 d.            -------- 96|0)
        4        5)     1201200 (24024|0 (250 l.
      ----              .......    ...
      715 qrs.          10        192
                        ---       ---
                         20        482
                         20        480
                        ---       ---
                          12  Rem. 240 qrs. or 5 s.
                          10
                         ---
                          20
                          20
        l.   s.          ---
Facit 250--5             (0)
```

Queſt. 12. A draper bought of a merchant 50 pieces of kerſey, each piece containing 34 ells Flemiſh (the ell Flemiſh being 3 quarters of a yard), to pay after the rate of 8 s. 4 d. *per* ell Engliſh; I demand how much the 50 pieces coſt him at that rate?

Firſt find out how many ells Flemiſh are in the 50 pieces; by multiplying 50 by 34, the product is 1700; which bring into quarters by 3, it makes 5100 quarters; then proceed as in the laſt queſtion, and the anſwer you will find to be 102000 pence, or 425 l. See the operation as followeth.

If

Qrs. *s.* *d.* *Qrs.*
If 5 : 8—4 : : 5100 50
12 100 34
100 d. 5) 510000 (102000 *d.* 20
15
5
1700 ells Flemiſh
10 3
10
5100
(0) 2|0 *l.*
12) 102000 (850|0 (425
96 8
60 5
60 4
(0) 10
10
Facit 425 l. (0)

Queſt. 13. A goldſmith bought a wedge of gold which weighed 14 lb. 3 oz. 8 pw. for the ſum of 514 l. 4 s. I demand what it ſtood him in *per* ounce? *Anſ.* 60 ſhillings, or 3 l. See the work.

lb. *oz.* *pw.* *l.* *s.* *oz.*
If 14—3—8 : 514—4 : : 1
12 20 20
31 10284 ſhillings 20 pw.
14 20
2|0)
171 oz. 3428) 205680 (6|0 (3 l.
20 6
20568 *s.* *l.*
3428 pw. (0) *Facit* 60 or 3
(0)

Queſt. 14. A grocer bought 4 hogſheads of ſugar, each weighing neat 6 C. 2 qrs. 14 lb. which coſt him 2 l. 8 s. 6 d. *per* C. I demand the value of the 4 hogſheads at that rate?

Firſt find the weight of the 4 hogſheads; which you may do by reducing the weight of one of them into pounds, and multiply them by 4, (the number of hogſheads), and they make 2968 lb.: then ſay, If 1 C. or 112 lb. coſt 2 l. 8 s. 6 d. what will 2968 lb. coſt? *Facit* 64 l. 5 s. 3 d. as by the operation.

```
                                                   C.   qrs.   lb.
                                                   6------2------14
                                                   4
                                                 -----
                                                   26
                                                   28
                                                 -----
 lb.       l.  s.  d.        lb.                  212
 112   :   2---8---6   : :   2968                  53
           20                 582                -----
          ----              -----                 742 lb in 1 hhd.
           48                5936                   4 hhds.
           12               23744                -----
          -----             14840                2968 l. in 4 hhds.
          102              --------        12)      2|0)
           48      112)  1727376   (15423   (128|5   (64 l.
          -----             .....     .....     ...
          582               112         12        12
                           -----       -----     -----
                            607         34         8
                            560         24         8
                           -----       -----     -----
                            473        102       5 ſhillings
                            448         96
                           -----       -----
                            257         63
                            224         60
                           -----       -----
                            336          3 pence
                            336
                           -----                        l.  s.  d.
                            (0)                  Facit 64---5---3
                             K
```

Queſt. 15. A draper bought of a merchant 8 packs of cloth, each containing 4 parcels, and each parcel 10 pieces, and each piece 26 yards, and gave after the rate of 4 l. 16 s. for 6 yards; now I deſire to know how much he gave for the whole? *Anſ.* 6656 l.

Firſt find out how many yards there were in the 8 packs, and by the following work you will find there are 8320 yards: then ſay, If 6 yards coſt 4 l. 16 s. what will 8320 yards coſt? *&c.*

8 packs
4
32 parcels
10
320 pieces
26
192
64
8320 yards

Yards. *l.* *s.* *Yards.*
If 6 : 4—16 : 8320
20 96

96 4992
7488

2|0)
6) 798720 (13312|0 (6656 l.

6 12
19 13
18 12
18 11
18 10
7 12
6 12
12 (0)
12
(0)

Facit 6656 l.

10. By this time the learner is, as I ſuppoſe, well exerciſed in the practic and theoretic of the rule of three direct; but at his leiſure he may look over the following queſtions, whoſe anſwers are given, but the operation purpoſely omitted, as a touchſtone for the learner, thereby to try his ability in what hath been delivered in the former rules.

Queſt. 16. If 24 lb. of raiſins coſt 6 s. 6 d. what will 18 frails coſt, each weighing neat 3 qrs. 18 lb. *Anſ.* 24 l. 17 s. 3 d.

Queſt. 17. If an ounce of ſilver be worth 5 ſhillings, what is the price of 14 ingots, each ingot weighing 7 lb. 5 oz. 10 pw.? *Anſ.* 313 l. 5 s.

Queſt. 18. If a piece of cloth coſt 10 l. 16 s. 8 d. I demand how many ells Engliſh there are in the ſame, when the ell at that rate is worth 8 s. 4 d. *Anſ.* 26 ells Engliſh.

Queſt. 19. A factor bought 84 pieces of ſtuffs, which coſt him in all 537 l. 12 s. at 5 s. 4 d. *per* yard, I demand how many yards there were in all, and how many ells Engliſh were contained in a piece of the ſame? *Anſ.* 2016 yards in all, and $19\frac{1}{5}$ ells Engliſh *per* piece.

Queſt. 20. A draper bought 242 yards of broad cloth, which coſt him in all 254 l. 10 s. for 86 yards, of which he gave after the rate of 21 s. 4 d. *per* yard, I demand how much he gave *per* yard for the remainder? *Anſ.* 20 s. $10\frac{64}{156}$ d. *per* yard.

Queſt. 21. A factor bought a certain quantity of ſerge and ſhalloon, which, together, coſt him 226 l. 14 s. 10 d. the quantity of ſerge he bought was 48 yards at 3 s. 4 d. *per* yard; and for every two yards of ſerge he had 5 yards of ſhalloon; I demand how many yards of ſhalloon he had, and how much the ſhalloon coſt him *per* yard? *Anſ.* 120 yards of ſhalloon at 1 l. 16 s. $5\frac{58}{120}$ d. *per* yard.

Queſt. 22. An oilman bought 3 tuns of oil, which coſt him 151 l. 14 s.; and ſo it chanced that it leaked out 85 gallons; but he is minded to ſell it again, ſo as that he may be no loſer by it, I demand how he muſt ſell it *per* gallon? *Anſ.* At 4 s. $6\frac{174}{671}$ d. *per* gallon.

Queſt. 23. A merchant bought 6 packs of cloth, each pack containing 12 cloths, which at 8 s. 4 d. *per* ell Flemiſh,

miſh, coſt 1080 l. I demand how many yards there were in each cloth? *Anſ.* 27 yards in each cloth.

Queſt. 24. A gentleman hath 536 l. *per annum,* and his expenſes are, one day with another, 18 s. 10 d. 2 qrs. I deſire to know how much he layeth up at the year's end? *Anſ.* 191 l. 3 s. 1 qr.

Queſt. 25. A gentleman expendeth daily, one day with another, 27 s. 10$\frac{1}{2}$ d. and at the year's end layeth up 340 l. I demand how much is his yearly income? *Anſ.* 848 l. 14 s. 4$\frac{1}{2}$ d.

Queſt. 26. If I ſell 14 yards for 10 l. 10 s. how many ells Flemiſh ſhall I ſell for 285 l. 17 s. 6 d. at that rate? *Anſ.* 504$\frac{2}{5}$ ells Flemiſh.

Queſt. 27. If 100 l. in 12 months gain 6 l. intereſt, how much will 75 l. gain in the ſame time, and at the ſame rate? *Anſ.* 4 l. 10 s.

Queſt. 28. If 100 l. in 12 months gain 6 l. intereſt, how much will it gain in 7 months at that rate? *Anſ.* 3 l. 10 s.

Queſt. 29. A certain uſurer put out 75 l. for 12 months, and received principal and intereſt 81 l. I demand at what rate *per cent.* he received intereſt? *Anſ.* at 8 *per cent.*

Queſt. 30. A grocer bought two cheſts of ſugar, the one weighed neat 17 C. 3 qrs. 14 l. at 2 lb. 6 s. 8 d. *per* C. the other weighed neat 13 C. 1 qr. 21 lb. at 4$\frac{1}{2}$ d. *per* lb. which he mingled together; now I deſire to know how much a C. weight of this mixture is worth? *Anſ.* 2 l. 4 s. 3 d. 2$\frac{1078}{4067}$ qrs.

Queſt. 31. Two men, *viz.* A and B, departed both from one place; the one goes eaſt, and the other weſt; the one travelleth 4 miles a-day, and the other 5 miles a-day; how far are they diſtant the 9th day after their departure? *Anſ.* 81 miles.

Queſt. 32. A flying every day 40 miles, is purſued the 4th day after by B, poſting 50 miles a-day: now the queſtion is, In how many days, and after how many miles travel will A be overtaken? *More's arithmetic, cap.* 8. *queſt.* 7.

Anſ. B overtakes him in 12 days, when they have travelled 600 miles.

11. The general and firſt effect of the rule of three

direct is contained in the definition of the ſame; that is, to find a fourth number in proportion to three numbers given, as hath been fully ſhewn in all the foregoing examples.

The ſecond effect is, by the price or value of one thing, to find the price or value of many things of like kind.

The third effect is, by the price or value of many things, to find the price of one; or by the price of many things, (the ſaid price being one), to find the price of many things of like kind.

The fourth effect is, by the price or value of many things, to find the price or value of many things of like kind.

The fifth effect is, thereby to reduce any number of monies, weights, or meaſures, the one ſort into the other, as in the rules of reduction contained in the eighth chapter foregoing. Examples of its various effects have been already given.

45 160

The proof of the rule of three direct.

12. The rule of three direct is thus proved, *viz.*

Multiply the 1ſt number by the 4th, and note the product; then multiply the 2d number by the 3d: and if this product is equal to the product of the 1ſt and 4th, then the work is rightly performed; otherwiſe, it is erroneous.

So the firſt queſtion of this chapter (whoſe anſwer or 4th number we found to be 18 s.) is thus proved, *viz.* The 1ſt number is 4; which multiplied by 18 (the 4th) produceth 72; and the 2d and 3d numbers are 12 and 6, which multiplied together, produce 72, equal to the product of the 1ſt and 4th; and therefore I conclude the work to be rightly performed.

Always obſerving, that if any thing remain after you have divided the product of the 2d and 3d numbers by the 1ſt, ſuch remainder, in proving the ſame, muſt be added to the product of the 1ſt and 4th numbers, whoſe ſum will be equal to the product of the 2d and 3d; the 2d number being of the ſame denomination with the 4th, and the 1ſt of the ſame denomination with the 3d.

So the fourth queſtion of this chapter being again repeated, *viz.* If 14 lb. of tobacco coſt 27 s. what will 478 lb. coſt at that rate? the anſwer (or 4th number) was 46 l. 1 s. 10 d. 1 qr. $\frac{2}{14}$: which is thus proved, *viz.* Bring the fourth number into farthings, and it makes 44249; which multiplied by the 1ſt number 14, produceth 619488, (the remainder 2 being added thereto.) Then, becauſe I reduce my 4th number into farthings, I reduce my 2d (*viz.* 27 s.) into farthings, and they are 1296; which multiplied by the 3d number 478, their product is 619488, equal to the product of the 1ſt and 4th numbers; wherefore I conclude the operation to be true. This is an infallible way to prove the rule of three direct; and it is deduced from the 12th ſection of the 9th chapter of this book.

Thus much concerning the ſingle rule of three direct; and I queſtion not, but that by this time the learner is ſufficiently qualified to reſolve any queſtion pertinent to this rule, not depending upon fractions or geometrical magnitudes. Thoſe that are deſirous to ſee the demonſtration of this rule, let them read the ſixth chapter of the ingenious Mr Kerſey's appendix to Mr Wingate's arithmetic; or the ſixth chapter of Mr Oughtred's incomparable *clavis mathematica:* by both which authors this rule is largely demonſtrated, being grounded upon the 16th propoſition of the ſixth book of Euclid's elements.

CHAP. XI.

The ſingle rule of Three inverſe.

1. THE golden rule, or rule of three inverſe, is, when there are three numbers given to find a fourth in proportion to the three given numbers, ſo as the fourth proceeds from the ſecond according to the ſame rate, reaſon, or proportion, that the firſt proceeds from the third. Or the proportion is,

As the third number is in proportion to the firſt, ſo is the ſecond to the fourth. *Alſted. math. lib. 2. cap. 14.*

So if the three numbers given were 8, 12, and 16, and it were required to find a fourth number in an inverted proportion to theſe; I ſay, that as 16 (the third number) is the double of the firſt term or number 8; ſo muſt 12 (the ſecond number) be the double of the fourth; ſo will you find the fourth term or number to be 6. And as, in the rule of three direct, you multiply the ſecond and third together, and divide their product for a fourth proportional number: ſo,

2. In the rule of three inverſe, you muſt multiply the ſecond term by the firſt (or firſt term by the ſecond) and divide the product thereof by the third term, ſo the quotient will give you the fourth term ſought in an inverted proportion. The ſame order being obſerved in this rule, as in the rule of three direct, for placing and diſpoſing of the given numbers, after your numbers are placed, in order to know whether your queſtion is to be reſolved by the rule direct or inverſe, obſerve the general rule following.

3. When your queſtion is ſtated, and your numbers orderly diſpoſed, conſider, in the firſt place, whether the fourth term, or number ſought, ought to be more or leſs than the ſecond term; which you may eaſily do: and if it is required to be more or greater than the ſecond term, then the leſſer extreme muſt be your diviſor; but if it require leſs, then the higheſt extreme muſt be your diviſor; (in this caſe the firſt and third numbers are called *extremes* in reſpect of the ſecond); and having found out your diviſor, you may know whether your queſtion belong to the rule direct or inverſe: for if the third term be your diviſor, then it is inverſe; but if the firſt term be your diviſor, then it is direct: As in the following queſtions.

Queſt. 1. If 8 labourers can do a certain piece of work in 12 days, in how many days will 16 labourers do the ſame? *Anſw.* In 6 days.

Having placed the numbers according to the ſixth rule of the tenth chapter, I conſider, that if 8 men can finiſh the work in 12 days, 16 men will do it in leſs or fewer days than 12; therefore the biggeſt ex-

lab. *days.* *lab.*
8 : 12 :: 16
8
16) 96 (6 days
96
(0) *Facit* 6 days.

treme muſt be the diviſor, which is 16; and therefore it is the rule of three inverſe: wherefore I multiply the firſt and ſecond numbers together, *viz.* 8 by 12, and their product is 96; which divided by 16, quotes 6 days for the anſwer. And in ſo many days will 16 labourers perform a piece of work, when 8 men can do it in 12 days.

Queſt. 2. If, when the meaſure (*viz.* a peck) of wheat coſt 2 ſhillings, the penny-loaf weighed (according to the ſtandard ſtatute, or law of England) 8 ounces, I demand how much it will weigh when the peck is worth 1 s. 6 d. according to the ſame rate or proportion? *Anſw.* 10 oz. 13 pw. 8 gr.

Having placed and reduced the given numbers according to the ſixth and ninth rules of the tenth chapter, I conſider, that at 1 s. 6 d. *per* peck, the penny-loaf will weigh more than at 2 s. *per* peck; for as the price decreaſeth, the weight increaſeth; and as the price increaſeth, ſo the weight diminiſhes: wherefore becauſe the third term requires more than the firſt, the leſſer extreme muſt be the diviſor, *viz.* 1 s. 6 d. or 18 d.: and having finiſhed the work, I find the anſwer to be 10 oz. 13 pw. 8 gr.; and ſo much will the penny-loaf weigh when the peck of wheat is worth 1 s. 6 d. according to the given rate of 8 ounces, when the peck is worth 2 ſhillings. The work is plain in the margin.

```
 s.        oz.          s. d.
 2    :    8     : :    1—6
12        24            12
——        ——            ——
24        32            18
          16
          ——  oz. pw. gr.
    18)  192 (10—13—8
          18
         ———
   Rem.   12
          20
         ———
    18)  240 (13 pw.
          18
         ———
          60
          54
          —
   Rem.    6
          24
         ———
    18)  144 (8 gr.
         144
         ———
         (0)
```

Queſt. 3. How many pieces of money or merchandiſe, at 20 s. *per* piece, are to be given or received for 240 pieces, the value or price of every piece being 12 ſhillings? *Anſ.* 144 pieces. For if 12 s. require 240 pieces, then 20 s. will require leſs; therefore the bigger extreme muſt be the diviſor, which is the third number. See the work.

s. *pieces.* *s.*
If 12 : 240 : : 20
12
——
48
24
——
2|0) 288|0 (144 pieces at 20 s. *per* piece
. . .
2
—
8
8
—
8
8
—
(0)

Queſt. 4. How many yards of 3 quarters broad, are required to double or be equal in meaſure to 30 yards, that are 5 quarters broad? *Anſ.* 50 yards. For ſay, If 5 quarters wide require 30 yards long, what length will 3 quarters broad require? Here I conſider that 3 quarters broad will require more yards than 30; for the narrower the cloth is, the more in length will go to make equal meaſure with a broader piece.

qrs. *long.* *qrs.*
5 : 30 : : 3
5
——
3) 150 (50 yards
.
15
—
(0)

Queſt. 5. At the requeſt of a friend, I lent him 200 l. for 12 months; he promiſing to do me the like courteſy at my neceſſity; but when I came to requeſt it of him, he could let me have but 150 l. : now I deſire to know how long I may keep this money to make plenary ſatisfaction for my former kindneſs to my friend ? *Anſ.* 16 months. I ſay, If 200 l. will require 12 months, what will 150 l. require ? 150 l. will require more time than 12 months ; therefore the leſſer extreme (*viz.* 150) muſt be the diviſor ; multiply and divide, and you will find the fourth inverted proportional to be 16. And ſo many months I ought to keep the 150 l. for ſatisfaction.

Queſt. 6. If for 24 s. I have 1200 lb weight carried 36 miles, how many miles ſhall 1800 lb. be carried for the ſame money ? *Anſ.* 24 miles.

Queſt. 7. If for 24 s. I have 1200 lb. weight carried 36 miles, how many pound weight ſhall I have carried 24 miles for the ſame money ? *Anſ.* 1800 lb. weight.

Queſt. 8. If 100 workmen in 12 days finiſh a piece of work or ſervice, how many workmen are ſufficient to do the ſame in 3 days ? *Anſ.* 400 workmen.

Queſt. 9. A colonel is beſieged in a town in which are 1000 ſoldiers, with proviſion of victuals only for 3 months ; the queſtion is, How many of his ſoldiers muſt he diſmiſs that his victuals may laſt the remaining ſoldiers 6 months ? *Anſ.* 500 he muſt keep, and diſmiſs as many.

Queſt. 10. If 20 l. worth of wine is ſufficient for the ordinary of 100 men, when the tun is ſold for 30 l. how many men will the ſame 20 l. worth ſuffice when the tun is worth 24 l. ? *Anſ.* 125 men.

Queſt. 11. How much pluſh is ſufficient to line a cloak, which hath in it 4 yards of 7 quarters wide, when the pluſh is but 3 quarters wide ? *Anſ.* $9\frac{1}{3}$ yards of pluſh.

Queſt. 12. How many yards of canvas that is ell wide, will be ſufficient to line 20 yards of ſay that is 3 quarters wide ? *Anſ.* 12 yards.

Queſt. 13. How many yards of matting that is two foot wide, will cover a floor that is 24 foot long and 20 foot wide ? *Anſ.* 240 foot.

Queſt. 14. A regiment of ſoldiers conſiſting of 1000, are to have new coats, and each coat to contain 2 yards 2 quarters of cloth, that is 5 quarters wide, and they are to be lined with ſhalloon that is 3 quarters wide, I demand how many yards of ſhalloon will line them? *Anſ.* $16666\frac{2}{3}$ quarters or $4166\frac{2}{3}$ yards.

Queſt. 15. A meſſenger makes a journey in 24 days when the day is 12 hours long, I deſire to know in how many days he will go the ſame when the day is 16 hours long? *Anſ.* In 18 days.

Queſt. 16. I borrowed of my friend 64 l. for 8 months, and he hath occaſion another time to borrow of me for 12 months, I deſire to know how much I muſt lend to make good his former kindneſs to me? *Anſ.* 42 l. 13 s. d.

4. The general and firſt effect of the rule of three inverſe, is contained in the definition of the ſame; that is, to find a fourth term in a proportion inverted to the three numbers given.

The ſecond effect is, by two prices or values of two ſeveral pieces of money or merchandiſes known, to find how many pieces of the one price are to be given for ſo many of the other; and conſequently to reduce and exchange one ſort of money or merchandiſe into another: or contrariwiſe, to find the price of any piece given to be exchanged in an inverted proportion.

The third effect is, by two different prices of a meaſure of wheat bought or ſold, and the weight of the loaf of bread made anſwerable to one of the prices of the meaſure given, to find out the weight of the ſame loaf anſwerable to the other price of the ſaid meaſure given: or contrariwiſe, by the two ſeveral weights of the ſame priced loaf, and the price of the meaſure of wheat anſwerable to one of thoſe weights given, to find out the other price of the meaſure anſwerable to the other weight of the ſame loaf.

The fourth effect is, by two lengths and one breadth of two rectangular planes known, to find out another breadth unknown; or by two breadths and one length given, to find out another length unknown, in an inverted proportion.

The fifth effect is, by two times, and a capital ſum of money borrowed or lent, to find out another capital ſum anſwerable to one of the given times; or otherwiſe, by two capital ſums, and a time anſwerable to one of them given, to find out a time anſwerable to the other capital ſum in reciprocal reaſon.

The ſixth effect is, by two different weights of carriage, and the diſtance of the place in miles or in leagues given, to find another diſtance in miles anſwerable to the ſame price of payment; or otherwiſe, by two diſtances in miles, and the weight anſwerable to one of the diſtances (being carried for a certain price) to find out the weight anſwerable to the other diſtance for the ſame price.

The ſeventh effect is, by two numbers of workmen, and the time anſwerable to one of the numbers of workmen given, to find out the time anſwerable to the other number of workmen in the performance of any work or ſervice: or contrariwiſe, by two times, and the workmen anſwerable to one of thoſe times given, to find out the number of workmen anſwerable to the other time, in the performance of any work or ſervice.

Alſo by two prices of proviſion, and the number of men or other creatures nouriſhed for a certain time anſwerable to one of the prices of proviſions given, to find out another number of men or other creatures anſwerable to the other price of the proviſion for the ſame time: or contrariwiſe, by two numbers of men or other creatures nouriſhed, and one price of proviſion anſwerable to one of the numbers of creatures given, to find out the other price of the ſame proviſion anſwerable to the other number of creatures, both being ſuppoſed to be nouriſhed for the ſame time, &c. as in the foregoing examples is fully declared.

To prove the operation of the rule of three inverſe, multiply the third and fourth terms together, and note their product; and multiply the firſt and ſecond together: and if their product is equal to the product of the third and fourth, then is the work truly wrought; but if it falleth out otherwiſe, then it is erroneous:

As in the firſt queſtion of this chapter, 16 (the third number) being multiplied by 6 (the fourth number), the product is 96; and the product of 8 (the firſt number) multiplied by 12 (the ſecond number) is 96, equal to the firſt product; which proves the work to be right.

And note, that if in diviſion any thing remain, ſuch remainder muſt be added to the product of the third and fourth terms; and if the ſum be equal to the product of the firſt and ſecond (the homogeneal terms being of one denomination), the work is right.

C H A P. XII.

The double Rule of Three direct.

WE have already delivered the rule of ſingle proportion, and we come now to lay down the rules of plural proportion.

1. Plural proportion is, when more operations in the rule of three than one are required before a ſolution can be given to the queſtion propounded: therefore in queſtions that require plurality in proportion, there are always given more than three numbers.

2. When there are given five numbers, and a ſixth is required in proportion thereunto, then this ſixth proportional is ſaid to be found out by the double rule of three, as in the queſtion following, *viz.*

If 100 l. in 12 months gain 6 l. intereſt, how much will 75 l. gain in 9 months?

3. Queſtions in the double rule of three, may be reſolved either by two ſingle rules of three, or by one ſingle rule of three compounded of the five given numbers.

4. The double rule of three, is either direct, or elſe inverſe.

5. The double rule of three direct is, when unto five given numbers a ſixth proportional may be found out by two ſingle rules of three direct.

6. The five given numbers in the double rule of three direct conſiſt of two parts, *viz.* firſt, a ſuppoſition; and, ſecondly, a demand: the ſuppoſition is contained in the three firſt of the five given numbers, and the demand lies in the two laſt: As in the example of the ſecond rule of this chapter, *viz.* If 100 l. in 12 months gain 6 l. intereſt, what will 75 l. gain in 9 months? Here the ſuppoſition is expreſſed in 100, 12, and 6; for it is ſaid, If 100 l. in 12 months gain 6 l. intereſt: and the demand lieth in 75

and 9; for it is demanded, how much 75 l. will gain in 9 months?

7. The next thing is to diſpoſe of the given numbers in due order and place, as a preparative for reſolution. Which that you may do, firſt, obſerve which of the given numbers in the ſuppoſition is of the ſame denomination with the number required; for that muſt be the ſecond number (in the firſt operation) of the ſingle rule of three; and one of the other numbers in the ſuppoſition (it matters not which) muſt be the firſt number; and that number in the demand which is of the ſame denomination with the firſt, muſt be the third number: which three numbers being thus placed, will make one perfect queſtion in the ſingle rule of three: As in the forementioned example, firſt, I conſider, that the number required in the queſtion, is the intereſt or gain of 75 l.; therefore that number in the ſuppoſition which hath the ſame name, *viz.* 6 l. which is the intereſt or gain of 100 l. muſt be the ſecond number in the firſt operation; and either 100 or 12 (it matters not which) muſt be the firſt number: but I will take 100; and then for the third number I put that number in the demand which hath the ſame denomination with 100, which is 75, (for they both ſignify pounds principal); and then the numbers will ſtand as you ſee in the margin.

100 : 6 :: 75

But if I had for the firſt number put the other number in the ſuppoſition, *viz.* 12, which ſignifies 12 months; then the third number muſt have been 9, which is the number in the demand which hath the ſame denomination with the firſt, *viz.* 9 months; and then they will ſtand as in the margin.

12 : 6 :: 9

100 : 6 :: 75
12 9

Or thus,

12 : 6 :: 9
100 75

There yet remain two numbers to be diſpoſed of; and thoſe are, one in the ſuppoſition, and another in the demand: that which is of the ſuppoſition, I place under the firſt of the three numbers; and the other, which is of the demand, I place under the third number; and then two of the terms in the ſuppoſition will ſtand one over the other in the firſt place, and the two terms in the demand will ſtand one over the other in the third place, as in the margin.

8. Having diſpoſed or ordered the given numbers according to the laſt rule, we may proceed to a reſolution. And, firſt, I work with the three uppermoſt numbers, which according to the firſt diſpoſition are 100, 6, and 75; which is as much as to ſay, If 100 l. require 6 l. intereſt, how much will 75 l. require? which by the third rule of the 11th chapter, I find to be direct, and by the ſeventh and eighth rules of the 10th chapter I find the fourth proportional number to be 4 l. 10 s.; ſo that by the foregoing ſingle queſtion I have diſcovered how much intereſt 75 l. will gain in 12 months; the operation whereof followeth, on the left hand under the letter A. And having diſcovered how much 75 l. will gain in 12 months, we may by another queſtion eaſily diſcover how much it will gain in 9 months; for this fourth number, thus found, I put in the middle between the two loweſt numbers of the five, after they are placed according to the ſeventh rule of this chapter; and then it will be the ſecond term in another queſtion of the rule of three, the numbers being
m. *l.* *s.* *m.*
12 : 4—10 : : 9, where the firſt and third numbers are of one denomination, *viz.* both months, and may be thus expreſſed: If 12 months require 4 l. 10 s. intereſt, what will 9 months require? and by the third rule of the 11th chapter, I find it to be the direct rule; and by working according to the directions laid down in the ſeventh, eighth, and ninth rules of the 10th chapter, I find the fourth proportional number to the laſt ſingle queſtion, to be 3 l. 7 s. 6 d. which is the ſixth proportional number to the five given numbers, and is the anſwer to the general queſtion. The work of the laſt ſingle queſtion is expreſſed on the right ſide of the page, under the letter B, as followeth.

l. *l.* *l.*
100 : 6 :: 75
A. m. 12 9 m. B.

l. *l.* *l.*
If 100 : 6 :: 75
75
——
30
42
—— *l.* *s.*
1|00 4|50 (4—10
4
——
Rem. 50
Mult. 20
——
1|00) 10|00 (10 s.

l. *s.*
Facit 4——10

Then ſay,
m. *l.* *s.* *m.*
If 12 : 4—10 :: 9
20
——
90 ſhillings
12
——
18
9
——
1080 pence
9
—— 12) 2|0 *l.* *s.* *d.*
12) 9720 (810 (6|7 (3—7—6
96 72 6
—— —— ——
12 90 7 s.
12 84
—— ——
(0) 6 pence

l. *s.* *d.*
Facit 3——7——6

So that, by the foregoing operation, I conclude, that if 100 l. in 12 months gain 6 l. intereſt, 75 l. will gain 3 l. 7 s. 6 d. in 9 months after the ſame rate. The anſwer would have been the ſame if the five given numbers had been ordered according to the ſecond method, *viz.* as you ſee in the margin.

12 : 6 :: 9
100 75

For, firſt, I ſay, If 12 months gain 6 l. what will 9 months gain? This queſtion I find to be direct by the third rule of the 11th chapter; and by the ſeventh and eighth rules of the 10th chapter, I find the fourth proportional number to theſe three to be 4 l. 10 s.

Thus I have found out what is the intereſt of 100 l. for 9 months; and I am now to find the intereſt of 75 l. for 9 months: to effect which, I make this fourth number (found as before) to be my ſecond number in the next queſtion, and ſay, If 100 l. require 4 l. 10 s. what will 75 l. require? This queſtion I find, by the ſaid third rule of the 11th chapter, to be direct; and by the ſaid ſeventh, eighth, and ninth rules of the 10th chapter, I find the anſwer to be as before, *viz.* 3 l. 7 s. 6 d.

This rule hath been ſufficiently explained by the foregoing example; ſo that the learner may be able to reſolve the following, or any other queſtions pertinent to the double rule of three direct, whoſe anſwers are here given; but the operations are purpoſely omitted, to try the learner's ability in the knowledge of what has been before delivered.

Queſt 2. A ſecond example in this rule may be as followeth, *viz.* A carrier receiving 42 ſhillings for the carriage of 3 C. weight 150 miles, I demand how much he ought to receive for the carriage of 7 C. 3 qrs. 14 l. 50 miles at that rate? *Anſ.* 36 s. 9 d.

Queſt. 3. If a regiment of 936 ſoldiers eat up 351 quarters of wheat in 168 days, I demand how many quarters of wheat 11232 ſoldiers will eat in 56 days at that rate? *Anſ.* 1404 quarters.

Queſt. 4. If 40 acres of graſs be mowed by 8 men in 7 days, how many acres ſhall be mowed by 24 men in 28 days? *Anſ.* 480 acres.

Queſt. 5. If 48 buſhels of corn, or other ſeed, yield 576 buſhels in 1 year, how much will 240 buſhels yield in 6 years at that rate; that is to ſay, if there were ſowed 240 buſhels every one of the 6 years? *Anſ.* 17280 buſhels.

Queſt. 6. If 40 ſhillings is the wages of 8 men for 5 days, what will be the wages of 32 men for 24 days? *Anſ.* 768 ſhillings, or 38 l. 8 s.

Queſt. 7. If 14 horſes eat 56 buſhels of provender in 16 days, how many buſhels will 20 horſes eat in 24 days? *Anſ.* 120 buſhels.

Queſt. 8. If 8 cannons in one day ſpend 48 barrels of powder, I demand how many barrels 24 cannons will ſpend in 12 days at that rate? *Anſ.* 1728 barrels.

Queſt. 9. If in a family conſiſting of 7 perſons, there are drunk out 2 kilderkins of beer in 12 days, how many kilderkins will there be drunk out in 8 days by another family conſiſting of 14 perſons? *Anſ.* 48 gallons, or 2 kilderkins and 12 gallons.

Queſt. 10. An uſurer put 75 l. out to receive intereſt for the ſame, and when it had continued 9 months, he received for principal and intereſt 78 l. 7 s. 6 d. I demand at what rate *per cent. per annum* he received intereſt? *Anſ.* at 6 *per cent. per annum.*

C H A P. XIII.

The double Rule of Three inverſe.

1. THE double rule of three inverſe is, when a queſtion in the double rule of three is reſolved by two ſingle rules of three, and one of thoſe ſingle rules falls out to be inverſe, or requires a fourth number in an inverted proportion; for both queſtions are never inverſe.

2. In all queſtions of the double rule of three, as well inverſe as direct, you are, in the diſpoſing of the five given numbers, to obſerve the ſeventh rule of the 12th chapter; and in reſolving of it by two ſingle rules, obſerve to make choice of your numbers for the firſt and ſecond ſingle queſtions according to the directions given in the eighth rule of the ſame chapter, as in the example following, *viz.*

Queſt. 1. If 100 l. principal in 12 months gain 6 l. intereſt, what principal will gain 3 l. 7 s. 6 d. in 9 months?

This queſtion is an inverſion of the firſt queſtion of the 12th chapter, and may ſerve for a proof thereof.

In order to a reſolution, I diſpoſe of the five given numbers according to the ſeventh rule of the laſt chapter; and being ſo diſpoſed, they will ſtand as *per* margin.

m.		*l.*		*m.*
12	:	100	::	9
l.				*l. s. d.*
6				3—7—6
		Or thus,		
l.		*l.*		*l. s. d.*
6	:	100	::	3—7—6
m. 12				9 *m.*

Here obſerve, that, according to the eighth rule of the 12th chapter, the firſt queſtion, if you take it from the five numbers, as they are ordered or placed firſt, will be, If 12 months require 100 l. principal, what will 9 months require to make the ſame intereſt? This, according to the third rule of the 11th chapter, is inverſe; and the anſwer will be found, by the ſecond rule of the 11th chapter, to be 133 l. 6 s. 8 d. The ſecond queſtion then will be, If 6 l. intereſt require 133 l. 6 s. 8 d. principal, how much principal will 3 l. 7 s. 6 d. require? This is a direct rule; and the anſwer in a direct proportion, is 75 l. See the work.

First I ſay

```
    m.       l.       m.
If 12   :   100   : : 9
            12
           ----   l.   s.  d.
      9)   1200 (133—6—8
           ...
            9                   l.   s.  d.
           --            Facit 133—6—8
           30
           27
           ----
            30
            27
           -----
            (3)
            20
            --
       9)   60 (6 s.
            54
           -----
            (6)
            12
            --
       9)   72 (8 d.
            72
            --
            (0)
```

Then I ſay,

l. : l. s. d. : : l. s. d.
If 6 : 133 — 6 — 8 : : 3 — 7 — 6

240 20 20

1440 d. 2666 67

12 12

5340 140

2666 67

32000 810 d.

810

32

256

24|0)

144|0) 2592000|0 (1800|0 d. or 75 l.

144 168

1152 120

1152 120

(0) (0)

So that by the foregoing work I find, that if 6 l. intereſt be gained by 100 l. in 12 months, 3 l. 7 s. 6 d. will be gained by 75 l. in 9 months.

But if the reſolution had been found out by the numbers as they are ranked in the ſecond place, then the ſecond queſtion in the ſingle rule would have been inverſe, and the firſt queſtion direct, and the concluſion the ſame with the firſt method, *viz.* 75 l.

Queſt. 2. If a regiment, conſiſting of 936 ſoldiers, can eat up 351 quarters of wheat in 168 days, how many ſoldiers will eat up 1404 quarters in 56 days at that rate? *Anſ.* 11232 ſoldiers.

Queſt. 3. If 12 ſtudents in 8 weeks ſpend 48 l. I demand how many ſtudents will ſpend 288 l. in 18 weeks? *Anſ.* 32 ſtudents.

Queſt. 4. If 48 l. ſerve 12 ſtudents 8 weeks, how many weeks will 288 l. ſerve 4 ſtudents? *Anſ.* 144 weeks.

Queſt. 5. If when a buſhel of wheat coſt 3 s. 4 d. the penny-loaf weigheth 12 ounces, I demand the weight of the loaf worth 9 d. when the buſhel coſt 10 s.? *Anſ.* 36 ounces.

Queſt. 6. If 48 pioneers in 12 days caſt a trench 24 yards long, how many pioneers will caſt a trench 168 yards long in 16 days? *Anſ.* 252 pioneers.

Queſt. 7. If 12 C. weight, being carried 100 miles, coſt 5 l. 12 s. I deſire to know how many C. weight may be carried 150 miles for 12 l. 12 s. at that rate? *Anſ.* 18 C.

Queſt. 8. If when wine is worth 30 l. *per* tun, 20 l. worth is ſufficient for the ordinary of 100 men, how many men will 4 l. worth ſuffice when it is worth 24 l. *per* tun? *Anſ.* 25 men.

Queſt. 9. If 6 men in 24 days mow 72 acres, in how many days will 8 men mow 24 acres? *Anſ.* in 6 days.

Queſt. 10. If when the tun of wine is worth 30 l. 100 men will be ſatisfied with 20 l. worth, I deſire to know what the tun is worth when 4 l. worth will ſatisfy 25 men at the ſame rate? *Anſ.* 24 l. *per* tun.

CHAP. XIV.

The Rule of Three compoſed of five Numbers.

1. THE rule of three compoſed, is, when queſtions, wherein there are five numbers given to find a ſixth in proportion thereunto, are reſolved by one ſingle rule of three compoſed of the five given numbers.

2. When queſtions may be performed by the double rule of three direct, and it is required to reſolve them by the rule of three compoſed; firſt order or rank your numbers according to the ſeventh rule of the 12th chapter; then

The rule is,

Multiply the terms or numbers that ſtand one over the other

other in the firſt place, the one by the other, and make their product the firſt term in the rule of three direct. Then multiply the terms that ſtand one over the other in the third place, and place their product for the third term in the rule of three direct, and put the middle term of the three uppermoſt for a ſecond term. Then having found a fourth proportional direct to theſe three, this fourth proportional ſo found ſhall be the anſwer required.

So the firſt queſtion of the 12th chapter being propoſed, *viz.* If 100 l. in 12 months gain 6 l. intereſt, what will 75 l. gain in 9 months? the numbers being ranked or placed as is there directed and done.

I multiply the two firſt terms, 100 and 12, the one by the other, and their product is 1200 for the firſt term. Then I multiply the two laſt terms 75 and 9 together, and their product is 675 for the third term. Then I ſay, As 1200 is to 6, ſo is 675 to the anſwer; which by the rule of three direct, will be found to be 3 l. 7 s. 6 d. as was before found.

3. But if the queſtion be to be anſwered by the double rule of three inverſe; then (having placed the five given terms as before) multiply the lowermoſt term of the firſt place, by the uppermoſt term of the third place, and put the product for the firſt term. Then multiply the uppermoſt term of the firſt place, by the lowermoſt term of the third place, and put the product for the third term; and put the ſecond term of the three higheſt numbers for the middle term to thoſe two. Then if the inverſe proportion is found in the uppermoſt three numbers, the fourth proportional direct to theſe three ſhall be the anſwer. So the firſt queſtion in the 13th chapter being ſtated, *viz.* If 100 l. principal in 12 months gain 6 l. intereſt, what principal will gain 3 l. 7 s. 6 d. in 9 months? ſtate the numbers as is there directed in the firſt order, as in the margin. Then reduce the 6 l. and 3 l. 7 s. 6 d. into pence; the 6 l. is 1440 d. and 3 l. 7 s. 6 d. is 810 d. Then multiply 1440 by 9, the product is 12960 for the firſt term in the rule of three direct; and multiply 810 by 12, the product

m.		*l.*		*m.*
12	:	100	: :	9
l.				*l.* *s.* *d.*
6				3 — 7 — 6

product is 9720 for the third term. Then ſay, As 12960 is to 100 l. ſo is 9720 to the anſwer, *viz.* 75 l. as before. But if the terms had been placed after the ſecond order, *viz.* as in the margin; then the inverſe proportion is found in the loweſt numbers; and having compoſed the numbers for a ſingle rule of three, as in the ſecond rule foregoing, then the anſwer muſt be found by a ſingle rule of three inverſe: for here it falls out to multiply 810 by 12 for the firſt number, and 1440 by 9 for the third number; and then you muſt ſay, As 9720 is to 100 l. ſo is 12960 to the anſwer; which by inverſe proportion will be found to be 75 l. as before.

l.		*l.*		*l.*	*s.*	*d.*
6	:	100	: :	3—	7—	6
m.				*m.*		
12				9		

The queſtions in the 12th and 13th chapters may ſerve for thy farther experience.

C H A P. XV.

Single Fellowſhip.

1. FEllowſhip is that rule of plural proportion, whereby we balance accounts depending between divers perſons having put together a general ſtock, ſo that they may every man have his proportional part of gain, or ſuſtain his proportional part of loſs.

2. The rule of fellowſhip, is either ſingle, or it is double.

3. The ſingle rule is, when the ſtocks propounded are ſingle numbers, without any reſpect or relation to time, each partner continuing his money in ſtock for the ſame time.

4. In the ſingle rule of fellowſhip, the proportion is, As the whole ſtock of all the partners is in proportion to the total gain or loſs, ſo is each man's particular ſhare in the ſtock to his particular ſhare in the gain or loſs. Therefore take the total of all the ſtocks for the firſt term in the rule of three, and the whole gain or loſs for the ſecond term, and the particular ſtock of any one of

the partners for the third term; then multiply and divide according to the ſeventh rule of the 9th chapter; and the fourth proportional number is the particular loſs or gain of him whoſe ſtock you made your third number: wherefore repeat the rule of three as often as there are particular ſtocks or partners in the queſtion; and the fourth terms produced upon the ſeveral operations, are the reſpective gain or loſs of thoſe particular ſtocks given; as in the examples following.

Queſt. 1. Two perſons, *viz.* A and B, bought a tun of wine for 20 l. of which A paid 12 l. and B paid 8 l. and they gained in the ſale thereof 5 l.; now I demand each man's ſhare in the gain according to his ſtock?

Firſt, I find the ſum of all their ſtocks, by adding them together, *viz.* 12 l. and 8 l. which are 20 l.; then, according to this rule, I ſay, firſt, If 20 l. (the ſum of their ſtock) require 5 l. (the total gain), how much will 12 l. (the ſtock of A) require? Multiply and divide by the ſeventh rule of the 9th chapter, and the anſwer is 3 l. for the ſhare of A in the gain. Then again I ſay, If 20 l. require 5 l. what will 8 l. require? The anſwer is 2 l. which is the gain of B. So I conclude, that the ſhare of A in the gain is 3 l. and the ſhare of B in the gain is 2 l. which in all is 5 l.

```
12
 8
--
20
```

```
      l.     l.      l.
If   20  :   5  ::  12
            12
           ---
     20)    60 (3 l.
            60
           ---
           (0)
```

```
      l.     l.      l.
If   20  :   5  ::   8
             8
           ---
     20)    40 (2 l.
```

Queſt. 2. Three merchants, *viz.* A, B, and C, enter upon a joint adventure; A put into the common ſtock 78 l. B put in 117 l. and C put in 234 l.; and they find, when they make up their accounts, that they have gained in all 264 l. ; now I deſire to know each man's particular ſhare in the gain?

Firſt, I add their particular ſtocks together, and their ſum is 429 l. ; then I ſay, If 429 l. gain 264 l. what will 78 l. gain? and what will 117 l. and what will 234 l. (the ſtocks of A, B, and C) gain? Work by three ſeveral rules of three, and you will find that

	l.
	78
	117
	234
Sum	429

The gain of		is	*l.*
	A		48
	B		72
	C		144
		Sum	264

Queſt. 3. Four partners, *viz.* A, B, C, and D, built a ſhip, which coſt 1730 l. of which A paid 346 l. B 519 l. C 692 l. and D 173 l. and her freight for a certain voyage is 370 l. which is due to the owners or builders; I demand each man's ſhare therein according to his charge in building her?

Anſwer		*l.*
	A	74
	B	111
	C	148
	D	37
	Sum	370

Queſt. 4. A, B, and C, enter into partnerſhip for a certain time; A put into the common ſtock 364 l. B put in 482 l. C put in 500 l. and they gained 867; now I demand each man's ſhare in the gain proportionable to his ſtock?

Anſwer		*l.*	*s.*	*d.*	*Rem.*
	A	234	9	3	354
	B	310	9	5	62
	C	322	1	3	930
	Sum	867	0	0	1346

The proof of the rule of ſingle fellowſhip.

5. To prove the rule of ſingle fellowſhip, add each man's particular gain or loſs together; and if the total ſum is equal to the general gain or loſs, then is the work rightly performed; but otherwiſe it is erroneous. Example: In the firſt queſtion of this chapter, the anſwer was, That the gain of A was 3 l. and the gain of B 2 l. which added together, makes 5 l. equal to the total gain given.

If in finding out the particular ſhares of the ſeveral partners any thing remain after diviſion is ended, ſuch remainders muſt be added together, (they being all fractions of the ſame denomination), and their ſum divided by the common diviſor in each queſtion, (*viz.* the total ſtock); and the quotient added to the particular gains; and then if the total ſum is equal to the total gain, the work is right, otherwiſe not.

As in the fourth queſtion, the remainders were 354, 62, and 930; which added together make 1346; which divided by 1346, (the ſum of their ſtocks), the quotient is 1 d.; which I add to the pence, *&c.* and the ſum of their ſhare is 867 l. equal to the total gain; wherefore I conclude the work is right.

C H A P. XVI.

Double Fellowſhip.

1. Double fellowſhip is, when ſeveral perſons enter into partnerſhip for unequal time; that is, when every man's particular ſtock hath relation to a particular time.

2. In the double rule of fellowſhip, multiply each particular ſtock by its reſpective time: and having added the ſeveral products together, make their ſum the firſt number or term in the rule of three, and the total gain or loſs the ſecond number, and the product of any one's particular ſtock by his time, the third term, and the fourth number

in proportion thereunto is his particular gain or loſs whoſe product of ſtock and time is your third number.

Then repeat, as in ſingle fellowſhip, the rule of three, as often as there are products or partners; and the fourth terms thereby invented, are the numbers required.

Examples.

Queſt. 1. A and B enter into partnerſhip; A put in 40 l. for 3 months, B put in 75 l. for 4 months, and they gained 70 l.; now I demand each man's ſhare in the gain proportional to his ſtock and time? *Anſ.* A 20 l. B 50 l.

To reſolve this queſtion, I firſt multiply the ſtock of A, viz. 40 l. by it time, 3 months, and the product is 120. Then I multiply the ſtock of B by its time, *viz.* 75 l. by 4, and it produceth 300; which I add to the product of A, his ſtock and time, and the ſum is 420. Then, by the rule of three direct, I ſay, As 420 (the ſum of the products) is to 70 (the total gain), ſo is 120 (the product of A his ſtock and time) to 20 l. (the ſhare of A in the gain.) Then I ſay again, As 420 is to 70, ſo is 300 to 50 l. (the ſhare of B in the gain.) And ſo much ought each to have for his ſhare.

l.	*l.*
40	75
3	4
A 120	B 300
	120
	Sum 420

Queſt. 2. A, B, and C, make a ſtock for 12 months; A put in at firſt 364 l. and 4 months after that he put in 40 l.; B put in at firſt 408 l. and at the end of 7 months he took out 86 l.; C put in at firſt 148 l. and 3 months after he put in 86 l. more, and 5 months after that he put in 100 l. more; and at the end of 12 months their gain is found to be 1436 l. I deſire to know each man's ſhare in the gain according to his ſtock and time?

Firſt, I conſider, that the whole time of their partnerſhip is 12 months; then I proceed to find out the ſeveral products of ſtock and time, as followeth.

A had at firſt 364 l. for 4 months, wherefore their product is, — — — — 1456

Then he put in 40 l. which, with the firſt ſum, makes 404 l. which continued the remainder of the time, *viz.* 8 months, and their product is, 3232

The ſum of the products of the ſtock and time of A is, — — — — — 4688

B had 408 l. in 7 months, whoſe product is 2856

And then took out 86 l.; therefore he left in ſtock 322 l. which continued the reſt of the time, *viz.* 5 months, whoſe product is, — — 1610

The ſum of the products of the ſtock and time of B is, — — — — — 4466

C put in 148 l. for 3 months, whoſe product is 444

Then he put in 86 l. which added to the firſt, *viz.* 148 l. makes 234 l. which lay in ſtock 5 months, their product is — — — 1170

Then he put in 100 l. more; ſo then he had in ſtock 334 l. which continued the remainder of the time, *viz.* 4 months, which multiplied together, produce — — — — 1336

The ſum of the product of the money and time of C is, — — — — — 2950
of B is, 4466
of A is, 4688
The total ſum of all the products is, — — 12104

Then I ſay, As 12104 is to 1436, the total gain; ſo is 2950 to the ſhare of C in the total gain, *&c.* Go on as in the foregoing examples, and you will ſind their ſhares in the gain to be as followeth, *viz.*

Anſwer,

		l.	*s.*	*d.*	*Rem.*
The ſhare of	A	556	3	6	6192
	B	529	16	9	5496
	C	349	19	8	416
Sum		1436	0	0	12104

Queſt. 3. Three graſiers, A, B, and C, take a piece of ground for 46 l. 10 s. in which A put 12 oxen for 8 months, B put in 16 oxen for 5 months, and C put in 18 oxen for 4 months; now the queſtion is, What each man ſhall pay of the 46 l. 10 s. for his ſhare in that charge?

Anſwer.

		l.	*s.*
A	ſhall pay	18	0
B		15	0
C		13	10
		46	10

3. The proof of this rule is the ſame with that of ſingle fellowſhip, laid down in the fifth rule of the 15th chapter. And note, that

If a loſs be ſuſtained inſtead of gain amongſt partners, every man's ſhare to be borne in the loſs is to be found after the ſame method as their gain, whether their ſtocks be for equal or unequal time.

CHAP. XVII.

Alligation Medial.

1. THE rule of alligation is that rule in plural proportion by which we reſolve queſtions wherein is a compoſition or mixture of divers ſimples; as alſo it is uſeful in compoſition of medicines, both for quantity, quality, and price. And its ſpecies are two, *viz.* medial and alternate.

2. Alligation medial is, when having the ſeveral quantities and prices of ſeveral ſimples propounded, we diſcover the mean price or rate of any quantity of the mixture compounded of thoſe ſimples. And the proportion is,

As the ſum of the ſimples to be mingled is to the total value of all the ſimples, ſo is any part or quantity of the compoſition or mixture to its mean rate or price.

Queſt. 1. A farmer mingled 20 buſhels of wheat, at 5 s. *per* buſhel, and 36 buſhels of rye, at 3 s. *per* buſhel, with 40 buſhels of barley, at 2 s. *per* buſhel; now I deſire to know what one buſhel of that mixture is worth?

To reſolve this queſtion, add together the given quantities, and alſo their values, which is 96 buſhels, whoſe total

total value is 14 l. 8 s. as appeareth by the work following. For,

Buſhels.	*l.*	*s.*
20 of wheat at 5 s. *per* buſhel, is	5	0
36 of rye at 3 s. *per* buſhel, is	5	8
40 of barley at 2 s. *per* buſhel, is	4	0
The ſum of their given quantities is, } 96, and their value is,	14	8

Then ſay, by the rule of three direct, If 96 buſhels coſt (or is worth) 14 l. 8 s. what is 1 buſhel worth?

Buſh.		*l.* *s.*		*Buſh.*
96	:	14——8	: :	1
		20		
	96)	288 (3 s		
		288 *Facit* 3 s. *per* buſh.		

(c)

Queſt. 2. A vintner mingleth 15 gallons of Canary, at 8 s. *per* gallon, with 20 gallons of Malaga, at 7 s. 4 d. *per* gallon, with 10 gallons of Malaga at 6 s. 10 d. *per* gallon, and 24 gallons of white-wine, at 4 s. *per* gallon; now I demand what a gallon of this mixture is worth? Work as in the laſt queſtion, and you will find the anſwer to be 6 s. 2 d. $3\frac{57}{69}$ qrs.

Queſt. 3. A grocer mingled 3 C. of ſugar at 56 s. *per* C. with 3 C. of ſugar at 3 l. 14 s. 8 d. *per* C. and with 6 C. at 1 l. 17 s. 4 d. *per* C. I deſire to know the price of a hundred weight of that mixture? *Anſ.* 2 l. 11 s. 4 d.

The proof of alligation medial.

3. The proof of this operation is, by the price of any quantity of the mixture, to find out the total value of the whole compoſition; and if it is equal to the total value of the ſeveral ſimples, the work is right; otherwiſe not: As in the firſt example, the anſwer to the queſtion was, That 3 s. is the price of 1 buſhel; wherefore I ſay, by the rule of proportion, If 1 buſhel be 3 s. what is 96

buſhels? *Anſ.* 14 l. 8 s. which is the total value of the ſeveral ſimples. Wherefore the work is right.

C H A P. XVIII.

Alligation Alternate.

1. ALligation alternate is, when there are given the particular prices of ſeveral ſimples, and thereby we diſcover ſuch quantities of thoſe ſimples, as being mingled together, ſhall bear a certain rate propounded.

2. When ſuch a queſtion is ſtated, place the given prices of the ſimples one over the other, and the propounded price of the compoſition againſt them, in ſuch ſort that it may repreſent a root, and they as ſo many branches ſpringing from it; as in the following example.

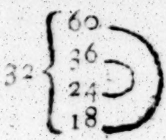
Queſt. 1. A certain farmer is deſirous to mix 20 buſhels of wheat at 5 s. or 60 d. *per* buſhel; with rye at 3 s. or 36 d. *per* buſhel, and with barley at 2 s. or 24 d. *per* buſhel, and oats at 1 s. 6 d. *per* buſhel; and deſireth to mix ſuch a quantity of rye, barley, and oats, with the 20 buſhels of wheat, as that the whole compoſition may be worth 2 s. 8 d. or 32 d. *per* buſhel.

The prices of the ſimples being placed according to the laſt rule, with the price of the compoſition propounded as a root to them, will ſtand as on the margin.

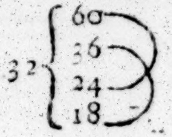
$$32\begin{cases}60 \text{ pence.}\\36\\24\\18\end{cases}$$

3. Having thus placed the given numbers, you are to link or combine the ſeveral rates of the ſimples the one to the other, by certain arches, in ſuch ſort, that one that is leſs than the root, or mean rate, may be linked or coupled to another that is greater than the mean rate. So the queſtion laſt propounded will ſtand,

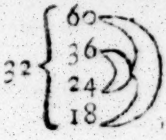
1. Thus,



2. Or thus,

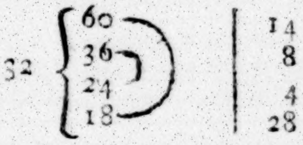


3. Or thus,



4. Then take the difference between the root and the ſeveral branches, and place the difference of each againſt the number or branch with which it is coupled or linked; and having taken all the differences, and placed them as aforeſaid, then thoſe differences ſo placed, will ſhew you the number of each ſimple to be taken to make a compoſition to bear the mean rate propounded.

So the branches of the laſt queſtion being linked together, as in the firſt manner, I ſay, The difference between 32 and 60 is 28; which I put againſt 18, becauſe 60 is linked with 18. Then the difference between 32 and 36 is 4; which I put againſt 24, becauſe 36 is linked or coupled with 24. Then I ſay, The difference between 32 and 24 is 8; which I place againſt 36, for the reaſon aforeſaid. Then I ſay, The difference between 32 and 18 is 14; which I place againſt 60. And then the work will ſtand as you ſee in the margin.



So I conclude, that a compoſition made of 14 buſhels of wheat at 60 d. *per* buſhel, and 8 buſhels of rye at 36 d. *per* buſhel, and 4 buſhels of barley at 24 d. *per* buſhel, and 28 buſhels of oats at 18 d. *per* buſhel, will bear the mean price of 32 d. or 2 s. 8 d. *per* buſhel. And here obſerve, that in the compoſition there is but 14 buſhels of wheat; but I would mingle 20 buſhels. And this kind, or rather caſe, of alligation alternate, *viz.* when there is given a certain quantity of one of the ſimples, and the quantities of the reſt ſought to mingle with this given quantity, that the whole may bear a price propounded, is called *alternation partial.*

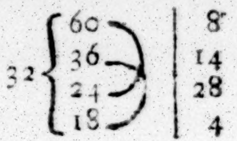
And the proportion to find out the ſeveral quantities to be mingled with the given quantity, is as followeth, *viz.*

As the difference annexed to the branch, that is the value of an integer of the given quantity, is to the other particular differences, ſo is the quantity given to the ſeveral quantities required.

So here, to find out how much rye, barley, and oats, muſt be mingled with the 20 buſhels of wheat, I ſay, by the ſingle rule of three direct, If 14 buſhels of wheat require 8 buſhels of rye, what will 20 buſhels of wheat require? *Anſ.* $11\frac{6}{14}$ buſhels of rye.

Again, If 14 buſhels of wheat require 4 buſhels of barley, what will 20 buſhels of wheat require? *Anſ.* $5\frac{10}{14}$ buſhels of barley. Again, I ſay, If 14 buſhels of wheat require 28 buſhels of oats, what will 20 buſhels of wheat require? *Anſ.* 40 buſhels of oats.

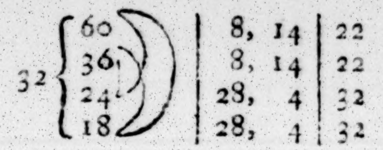
And now I ſay, That 20 buſhels of wheat mingled with $11\frac{6}{14}$ buſhels of rye, and $5\frac{10}{14}$ buſhels of barley, and 40 buſhels of oats, each bearing the rate as aforeſaid, will make a compoſition or heap of corn that may yield 32 d. *per* buſhel.



But if the branches had been coupled according to the ſecond order or manner, the differences would have been thus placed, *viz.* the difference between 32 and 60 is 28; which I ſet againſt 24, becauſe 60 is linked thereto: and the difference between 32 and 36 is 4; which I ſet againſt 18: and the difference between 32 and 24 is 8; which I ſet againſt 60: then the difference between 32 and 18 is 14; which I ſet againſt its yoke-fellow 36: and then I conclude, that if you mix 8 buſhels of wheat with 14 buſhels of rye, 28 buſhels of barley, and 4 buſhels of oats, each bearing the aforeſaid prices, the whole mixture may be ſold for 32 d. *per* buſhel; as by the work in the margin.

You ſee by this work we have found how many buſhels of rye, barley, and oats, ought to be mixed with 8 buſhels of wheat. And to find out how many of each ought to be mixed with 20 buſhels of wheat, I ſay, As 8 is to 14, ſo is 20 to 35 buſhels of rye; as 8 is to 28, ſo is 20 to 70 buſhels of barley; as 8 is to 4, ſo is 20 to 10 buſhels of oats: whereby I conclude, that if to 20 buſhels of wheat I put 35 buſhels of rye, 70 buſhels of barley, and 10 buſhels of oats, bearing each the aforeſaid price *per* buſhel, that then a buſhel of this mixture will be worth 32 d. or 2 s. 8 d.

And if the branches had been linked as you ſee in the third place, where each branch bigger than the root is linked to two that are leſs than the root; then in this caſe you muſt have placed the ſeveral differences between the root and branches, againſt thoſe two with which each is coupled: As firſt, the difference between 32 and 60 is 28; which I ſet againſt 24 and 18, becauſe it is coupled with them both. Then the difference between 32 and 36 is 4; which I ſet likewiſe againſt 24 and 18, becauſe 36 is linked to them both. Then the difference between 32 and 24 is 8; which I put againſt 60 and 36, becauſe 24 is linked to them both. Then the difference between 32 and 18 is 14; which I put againſt 60 and 36, the yoke-fellows of 18.



Laſtly, I draw a line behind the differences, and add the differences which ſtand againſt each branch, and put the ſum behind the ſaid line, againſt its proper branch; as you ſee in the margin.

And now, by this work, I find, that 22 buſhels of wheat mingled with 22 buſhels of rye, and 32 buſhels of barley, and 32 buſhels of oats, each bearing the ſaid price, will make a mixture bearing the mean rate of 32 d. *per* buſhel.

And to find how much of each of the reſt muſt be mingled with 20 buſhels of wheat, I ſay,

As 22 is to 22, ſo is 20 to 20 buſhels of rye; as 22 is to 32, ſo is 20 to $29\frac{2}{22}$ buſhels of barley; as 22 is to 32, ſo is 20 to $29\frac{2}{22}$ buſhels of oats.

Whereby you ſee that queſtions of alligation alternate will admit of more true anſwers than one; for we have found three ſeveral anſwers to this firſt queſtion.

The proof of alternation partial.

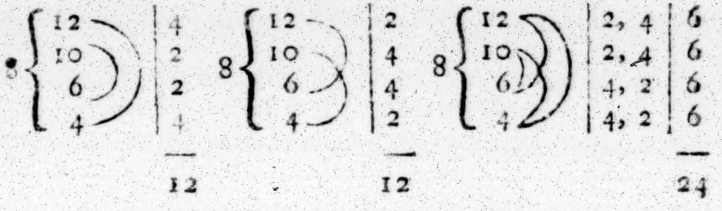
Queſtions of alternation partial, are proved the ſame way with queſtions in alligation medial; which you may ſee in the third rule of the 17th chapter.

Queſt. 2. A grocer hath four ſorts of ſugar, *viz.* of 12 d. *per* lb. of 10 d. *per* lb. of 6 d. *per* lb. and of 4 d. *per* lb. and he would have a compoſition worth 8 d. *per* lb. the whole quantity whereof ſhould contain 144 lb. made of theſe four ſorts; I demand how much of each he muſt take?

Queſtions of this nature are reſolved by that part of alligation alternate, called by arithmeticians, *alternation total, viz.* where there is given the ſum and prices of ſeveral ſimples, to find out how much of each ſimple ought to be taken to make the ſaid ſum or quantity, ſo that it may bear a certain rate propounded.

To reſolve this queſtion, I place the ſeveral prices of the ſimples and mean rate propounded, and link them together as is directed in the ſecond and third rules of this chapter, and place the differences between the root and branches, according to the fourth rule of this chapter; which will then ſtand one of theſe three ways, *viz.*

First. Second. Third.



5. Then add the ſeveral differences together, which I have done, and the ſums of the firſt and ſecond order are 12 lb. and of the third 24 lb. as you may ſee above. But it is required that there ſhould be 144 lb. of the compoſition; therefore, to find the quantity of each ſimple to make the whole compoſition 144 lb. obſerve this general rule, *viz.*

As the ſum of the differences is to the ſeveral differences, ſo is the total quantity of the compoſition to the quantity of each ſimple.

So to find how much of each ſort of ſugar I ought to take to make 144 lb. at 8 d *per* lb. I ſay,

As 12 is to 4, ſo is 144 to 48 lb. at 12 d. *per* lb.
As 12 is to 2, ſo is 144 to 24 lb. at 10 d. *per* lb.
As 12 is to 2, ſo is 144 to 24 lb. at 6 d. *per* lb.
As 12 is to 4, ſo is 144 to 48 lb. at 4 d. *per* lb.

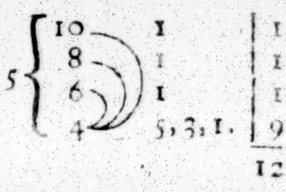
Whereby I find, that 48 lb. at 12 d. *per* lb. and 24 lb. at 10 d. *per* lb. and 24 lb. at 6 d. *per* lb. and 48 lb. at 4 d. *per* lb. will make a compoſition of ſugar containing 144 lb. worth 8 d. *per* lb.

But as the branches are linked in the ſecond order, the anſwer will be 24 lb. at 12 d. *per* lb. and 48 lb. at 10 d. *per* lb. and 48 lb. at 6 d. *per* lb. and 24 lb. at 4 d. *per* lb. to make the ſaid quantity, and to bear the ſaid price.

And if you had worked as the branches are linked in the third order, then you would have found the quantity of each to have been 36 lb.

Queſt. 3. A vintner hath four ſorts of wine, *viz.* Canary at 10 s. *per* gallon, Malaga at 8 s. *per* gallon, Rheniſh wine at 6 s. *per* gallon, and white-wine at 4 s. *per* gallon, and he is minded to make a compoſition of them all of 60 gallons, that may be worth 5 s. *per* gallon; I deſire to know how much of each he muſt have?

The number of terms being ranked according to the ſecond rule of this chapter, the branches will be linked as followeth; but will admit of no other manner of coupling, becauſe there is but one branch that is leſs than the root; therefore all the reſt muſt be linked unto it; and the differences between the root and the three firſt branches, *viz.* 10, 8, and 6, which are 5, 3, and 1, muſt be ſet againſt 4, becauſe they are coupled with it; and the difference between the root, *viz.* 5, and 4, which is 1, muſt be ſet againſt the three other, becauſe it is linked to them all: ſo I find 1 gallon of Canary, 1 gallon of Malaga, 1 gallon of Rheniſh wine, and 9 gallons of white-wine, prized as above, being mingled together, will be worth 5 s. *per* gallon, the ſum being 12 gallons. But there muſt be 60 gallons; wherefore I ſay,



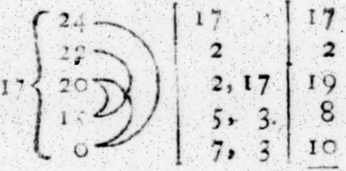
As 12 is to 1, ſo is 60 to 5 gallons of Canary.
As 12 is to 1, ſo is 60 to 5 gallons of Malaga.
As 12 is to 1, ſo is 60 to 5 gallons of Rheniſh.
As 12 is to 9, ſo is 60 to 45 gallons of white-wine.

So that 5 gallons of Canary, 5 gallons of Malaga, 5 gallons of Rheniſh, and 45 gallons of white-wine, mingled together, will be in all 60 gallons, worth 5 s. *per* gallon; which was required.

Queſt. 4. A goldſmith hath gold of four ſeveral ſorts of fineneſs, *viz.* of 24 careċts fine, and of 22 careċts fine, of

20 carects fine, and of 15 carects fine, (ſee chap. 2. def. 2. of this book), and he would mingle ſo much of each with alloy, that a maſs of 28 ounces of gold, ſo mingled, may bear 17 carects fine; I demand how much of each he muſt take? The ſecond and third rule of this chapter being obſerved, (inſtead of the alloy I put 0, becauſe it bears no fineneſs, but it makes a branch in the operation), the terms may be alligated, and the differences taken, by any of theſe four ways following, *viz.*

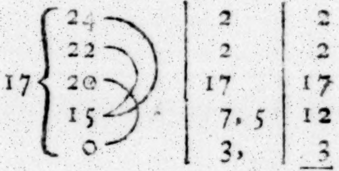
Firſt thus,



17	24	17	17
	22	2	2
	20	2, 17	19
	15	5, 3	8
	0	7, 3	10

Sum 56

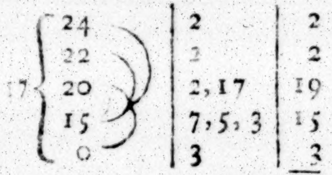
Secondly thus,



17	24	2	2
	22	2	2
	20	17	17
	15	7, 5	12
	0	3,	3

Sum 36

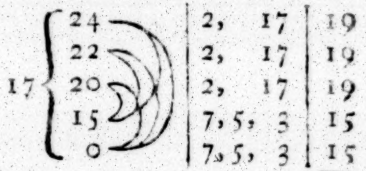
Thirdly thus,



17	24	2	2
	22	2	2
	20	2, 17	19
	15	7, 5, 3	15
	0	3	3

Sum 41

Fourthly thus,



17	24	2, 17	19
	22	2, 17	19
	20	2, 17	19
	15	7, 5, 3	15
	0	7, 5, 3	15

Sum 87

More ways may be given for the alligating or linking of the terms in this queſtion, but theſe are ſufficient for the induſtrious. And it ſhall alſo ſuffice to give an anſwer to the queſtion, as the terms are linked the firſt way, not doubting but the ingenious practitioner will be able at his leiſure to find anſwers to the other three ways, *viz.*

	oz.	*pw.*	*car.*
As 56 is to 17, ſo is 28 to	8——	10 of	24
As 56 is to 2, ſo is 28 to	1——	0 of	22
As 56 is to 19, ſo is 28 to	9——	10 of	20
As 56 is to 8, ſo is 28 to	4——	0 of	15
As 56 is to 10, ſo is 28 to	5——	0 of	alloy.

Thus much well practiſed and underſtood, is ſufficient for underſtanding alligation.

The proof of alternation total.

In queſtions of alternation total, the anſwer is given true, when the ſum or quantity of the ſimples found agrees with the ſum or quantity propounded; as in the laſt queſtion, the anſwer was 8 oz. 10 pw. of 24 carects fine, 1 oz. of 22 carects fine, 9 oz. 10 pw. of 20 carects fine, 4 oz. of 15 carects fine, and 5 oz. of alloy; which added together, makes 28 oz. the quantity propounded.

CHAP. XIX.

Reduction of Vulgar Fractions.

1. WHat a vulgar fraction is, and its parts and ſeveral kinds, hath been already ſhewed in the 19th, 20th, 21ſt, 22d, 23d, 24th, and 31ſt definitions of the firſt chapter of this book; which the learner is deſired diligently to obſerve before he proceeds.

2. How to reduce a vulgar fraction, we ſhall teach under theſe eight ſeveral heads, or rules, following, *viz.*

1. To reduce a mixed number into an improper fraction.
2. To reduce a whole number into an improper fraction.
3. To reduce an improper fraction into its equivalent whole or mixed number.
4. To reduce a fraction into its loweſt terms equivalent to the fraction given.
5. To find the value of a fraction in the known parts of coin, weight, meaſure, *&c.*
6. To reduce a compound fraction to a ſimple one of the ſame value.
7. To reduce divers fractions having unequal denominators, to fractions of the ſame value having equal denominators.
8. To reduce a fraction of one denomination to another of the ſame value.

I. *To reduce a mixed number to an improper fraction.*

The rule is, Multiply the integer-part, or whole number by the denominator of the fraction, and to the product add the numerator, and that ſum place over the denominator for a new numerator; ſo this new fraction ſhall be equal to the mixed number given. *Vid. cap.* 1. *defin.* 31. and 23. As for example:

1. Reduce $18\frac{3}{7}$ into an improper fraction. Multiply the whole number 18 by 7 the denominator, and to the product add the numerator 3, the ſum is 129; which put over the denominator 7, and it makes $\frac{129}{7}$ for the anſwer, as on the margin.

$$\begin{array}{r} 18\frac{3}{7} \\ 7 \\ \hline 129 \end{array}$$

Facit $\frac{129}{7}$

2. Reduce $183\frac{5}{12}$ to an improper fraction. *Facit* $\frac{2201}{12}$

3. Reduce $50\frac{13}{21}$ to an improper fraction. *Facit* $\frac{1063}{21}$

II. *To reduce a whole number into an improper fraction.*

The rule is, Multiply the given number by the intended denominator, and place the product for the numerator over it. As for example:

1. Let it be required to reduce 15 into a fraction, whoſe denominator ſhall be 12. To effect which, I multiply 15 by the intended denominator 12, the product is 180; which I place over 12 as a numerator, and it makes $\frac{180}{12}$, which is equal to 15, as was required, as *per* margin.

$$\begin{array}{r} 15 \\ 12 \\ \hline 30 \\ 15 \\ \hline 180 \end{array}$$

Facit $\frac{180}{12}$

2. Reduce 36 into an improper fraction, whoſe denominator ſhall be 26. *Facit* $\frac{936}{26}$.

3. Reduce 135 into an improper fraction, whoſe denominator ſhall be 16. *Facit* $\frac{2160}{16}$.

III. *To reduce an improper fraction into its equivalent whole or mixed number.*

The rule is, Divide the numerator by the denominator, and the quotient is the whole number equal to the fraction; and if any thing remain, put it for a numerator over the diviſor. As for example:

1. Reduce $\frac{436}{8}$ into its equivalent mixed number. Divide the numerator 436 by the denominator 8, and the quotient is 54, and 4 remains; which put for a numerator over the diviſor 8, the anſwer is $54\frac{4}{8}$; as in the margin.

$$\begin{array}{r} 8)\ 436\ (54 \\ 40 \\ \hline 36 \\ 32 \\ \hline (4) \end{array}$$

Facit $54\frac{4}{8}$

2. Reduce $\frac{1476}{15}$ to a mixed number. *Facit* $98\frac{6}{15}$.
3. Reduce $\frac{2357}{136}$ to a mixed number. *Facit* $173\frac{48}{136}$.

IV. *To reduce a fraction into its loweſt terms equivalent to the fraction given.*

The rule is, 1. If the numerator and denominator are even numbers, take half the one and half the other, as often as may be; and when either of them falls out to be an odd number, then divide them by any number that you can diſcover will divide both numerator and denominator without any remainder; and when you have thus proceeded as low as you can reduce them, then this new fraction ſo found out, ſhall be the fraction you deſire, and will be in value equal to the given fraction.

$\frac{192}{336}\Big|\frac{96}{168}\Big|\frac{48}{84}\Big|\frac{24}{42}\Big|\frac{12}{21}\Big|\frac{4}{7}$

Example 1. Let it be required to reduce $\frac{192}{336}$ into its loweſt terms. Firſt, I take the half of the numerator 192, and it is 96; then half of the denominator, and it is 168; ſo that now it is brought to $\frac{96}{168}$, and next to $\frac{48}{84}$, and by halving ſtill, to $\frac{24}{42}$, and their half is $\frac{12}{21}$; and now I can no longer half it, becauſe 21 is an odd number, wherefore I try to divide them by 3, *&c.* and I find 3 divides them both without any remainder, and brings them to $\frac{4}{7}$, as *per* margin. So I conclude $\frac{4}{7}$ thus found, to be equal in value to the given fraction $\frac{192}{336}$.

2. What is $\frac{1036}{1184}$ in its loweſt terms? *Anſ.* $\frac{7}{8}$.
3. What is $\frac{1342}{1586}$ in its loweſt terms? *Anſ.* $\frac{11}{13}$.

There is yet another way more excellent than the former, to reduce a fraction into its loweſt terms; and that is, 2. by finding a common meaſurer, *viz.* the greateſt number that will divide the numerator and denominator without any remainder, and by that means reduce a fraction to its loweſt terms at the firſt work. And to find out this common meaſurer, divide the denominator by the numerator; and if any thing remains, divide your diviſor thereby; and if any thing yet remain, then divide your laſt diviſor by it. Do ſo until you find nothing remaining; then this laſt diviſor ſhall be your greateſt common meaſurer, which will divide both numerator and denominator, and reduce them both into their loweſt terms at one work. *Vid. Ought. Cla. Math. cap.* 7.

Example 4. Reduce $\frac{228}{304}$ into its loweſt terms by a common meaſurer. To effect which, I divide the denominator 304, by the numerator 228, and there remains 76. Then I divide 228 (the firſt diviſor) by 76 (the remainder), and it quotes 3, and nothing remains. Wherefore the laſt diviſor 76 is the common meaſurer; by which I divide the numerator of the given fraction, *viz.* 228, it quotes 3 for a new numerator. Then I divide the denominator 304 by 76, and it quotes 4 for a new denominator. So that now I have found $\frac{3}{4}$ equal to $\frac{228}{304}$.

5. Reduce $\frac{6048}{8736}$ into its loweſt terms by a common meaſurer. *Facit* $\frac{9}{13}$.

6. Reduce $\frac{3081}{20145}$ into its loweſt terms by a common meaſurer. *Facit* $\frac{13}{85}$.

A compendium.

Note, That if the numerator and denominator of a fraction end each with a cipher or ciphers, then cut off as many ciphers from the one as from the other, and the remaining figures will be a fraction of the ſame value, thus, $\frac{3400}{7100}$ will be found to be reduced to $\frac{34}{71}$, by cutting off the two ciphers from the numerator and denominator with a daſh of the pen, thus, $\frac{34|00}{71|00}$; and $\frac{4600}{7500}$, will be $\frac{46}{75}$, thus, $\frac{46|00}{75|00}$, &c.

V. *To find the value of a fraction in the known parts of coin, weight,* &c.

The rule is, Multiply the numerator by the parts of the next inferior denomination that are equal to an unit of the ſame denomination with the fraction, then divide that product by the denominator, and the quote gives you its value in the ſame parts you multiplied by; and if any thing remain, multiply it by the parts of the next inferior denomination, and divide as before. Do ſo till you can bring it no lower, and the ſeveral quotients will give you the value of the fraction required; and if any thing at laſt remain, place it for a numerator over the former denominator. Some few examples will make the rule plain.

1. What is the value of $\frac{27}{29}$ l. Sterling? To anſwer this queſtion, I multiply the numerator 27 by 20 (the ſhillings in a pound), the product is 540; which I divide

by 29, (the denominator), and the quotient is 18 s. and there remains 18; which I multiply by 12, and the product (216) I divide by the denominator 29, the quotient is 7 d. and 13 remains; which I multiply by 4, the product is 52; which I ſtill divide by 29, the quotient is 1 farthing, and there remaineth 23; which I put for a numerator over the denominator 29: ſo I find the value of $\frac{27}{29}$ l. to be 18 s. 7 d. 1 qr. $\frac{23}{29}$, as by the operation on the margin. And after the ſame manner are the values of the fractions in the ſeveral examples following found out.

		l.
	27	$\frac{27}{29}$
Multiply	20	
		qrs.
29)	540	(18 s. 7 d. 1 $\frac{23}{29}$
	29	
	250	
	232	
Remains	18	
Multiply	12	
	36	
	18	
29)	216	(7 d.
	203	
Remains	13	
Multiply	4	qr
29)	52	(1 $\frac{23}{29}$
	29	
Remains	23	
	s. *d.* *qr.*	
Facit	18—7—1 $\frac{23}{29}$	

2. What is the value of $\frac{11}{15}$ l. Sterling? *Facit* 14 s. 8 d.

3. What is the value of $\frac{28}{137}$ l. Sterling? *Facit* 4 s. 1 d. $\frac{7}{137}$.

4. What is $\frac{16}{21}$ C. weight? *Facit* 3 qrs. 1 lb. 5 oz. $\frac{7}{21}$.

5. What is $\frac{136}{371}$ lb. Troy weight? *Facit* 4 oz. 7 pw. 23 gr. $\frac{179}{371}$.

6. What is $\frac{41}{50}$ of a year? *Anſ.* 299 days, 7 hours, 12 min.

VI. *To reduce a compound fraction to a ſimple one of the ſame value.*

What a compound fraction is hath been ſhewn in chap. 1. defin. 24.; and to reduce it to a ſimple fraction of the ſame value,

The rule is, Multiply the numerators continually, and place the laſt product for a new numerator; then multiply

tiply the denominators continually, and place the laſt product for a new denominator: ſo this ſingle fraction ſhall be equal to the compound fraction. For example:

1. Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a ſimple fraction.

Multiply the numerators 2, 3, and 5 together, they make 30 for a new numerator; then multiply the denominators 3, 5, and 8 together, and their product is 120 for a denominator: ſo the ſimple fraction is $\frac{30}{120}$, and cutting off the ciphers, it is $\frac{3}{12}$, equal to $\frac{1}{4}$ by the 4th rule; as in the margin.

5	3
3	2
—	—
15	6
8	5
—	—
120	30

Facit $\frac{30}{120}$, or $\frac{3}{12}$, or $\frac{1}{4}$.

2. What is $\frac{7}{10}$ of $\frac{5}{8}$ of $\frac{4}{7}$ of $\frac{11}{12}$? *Anſ.* $\frac{1540}{6720}$, or $\frac{154}{672}$, or $\frac{77}{336}$ in its loweſt terms.

3. What is $\frac{11}{12}$ of $\frac{13}{14}$ of $\frac{21}{20}$? *Anſ.* $\frac{3003}{4871}$.

By this you may know how to find the value of a compound fraction, *viz.* firſt reduce it to a ſimple one, and then find out its value by the fifth rule foregoing. As,

4. What is the value of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{9}{12}$ of a pound? *Anſ.* 9 s. 4 d. 2 qrs.

VII. *To reduce fractions of unequal denominators, to fractions of the ſame value having equal denominators.*

The rule is, Multiply all the denominators together, and the product ſhall be the common denominator. Then multiply each numerator into all the denominators, except its own, and the laſt product put for a numerator over the denominator, found out as before: ſo this new fraction is equal to that fraction whoſe numerator you multiplied into the ſaid denominators. Do ſo by all the numerators given, and you have your deſire. For example:

1. Reduce $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common denominator. Multiply the denominators 4, 5, 6, and 8, together continually; and put the product, 960, for the common denominator. Then multiply the numerator 3 into the denominators 5, 6, and 8, and the product is 720, which is a numerator to 960 (found as before). So $\frac{720}{960}$ is equal to the firſt fraction $\frac{3}{4}$. Then I proceed to find a new numerator to the ſecond fraction, *viz.* $\frac{4}{5}$, and I mul-

tiply 4 into all the denominators except its own, *viz.* into 4, 6, and 8, which produceth $\frac{768}{960}$ equal to $\frac{4}{5}$. Then I multiply the numerator 5 into the denominators 4, 5, and 8; the product is $\frac{800}{960}$, equal to $\frac{5}{6}$. Then I multiply the numerator 7 into the denominators 4, 5, and 6; the product is $\frac{840}{960}$, equal to $\frac{7}{8}$: and the work is done. So that for $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$, I have $\frac{720}{960}$, $\frac{768}{960}$, $\frac{800}{960}$, and $\frac{840}{960}$.

2. Reduce $\frac{11}{12}$, $\frac{24}{25}$, and $\frac{19}{21}$, into a common denominator. *Faciunt* $\frac{5770}{6300}$, $\frac{6048}{6300}$, $\frac{5700}{6300}$.

VIII. *To reduce a fraction of one denomination to another.*

This is either aſcending or deſcending: aſcending, when a fraction of a ſmaller is brought to a greater denomination; deſcending, when a fraction of a greater denomination is brought to a lower.

1. When a fraction is to be brought from a leſſer to a greater denomination, then make of it a compound fraction, by comparing it with the intermediate denominations between it and that you would have it reduced to: then (by the 6th rule foregoing) reduce your compound to a ſimple fraction, and the work is done. For example,

Queſt. 1. It is required to know what part of a pound Sterling $\frac{5}{7}$ of a penny is?

To reſolve this queſtion, I conſider that 1 d. is $\frac{1}{12}$ of a ſhilling, and a ſhilling is $\frac{1}{20}$ of a pound; wherefore, $\frac{5}{7}$ d. is $\frac{5}{7}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound; which, by the ſaid 6th rule, I find to be $\frac{5}{1680}$ of a pound Sterling, or Engliſh money.

Queſt. 2. What part of a pound Troy weight is $\frac{4}{5}$ of a penny-weight? *Anſ.* $\frac{4}{5}$ of $\frac{1}{20}$ of $\frac{1}{12}$ lb. equal to $\frac{4}{1200}$ lb. Troy.

2. When a fraction is to be brought from a greater to a leſſer denomination, then multiply the numerator by the parts contained in the ſeveral denominations betwixt it and the parts you would reduce it to; then place the laſt product over the denominator of the given fraction. For example:

Queſt. 3. I would reduce $\frac{3}{5}$ l. to the fraction of a penny; to do which, I multiply the numerator 3 by 20 and 12, the product is 720; which I put over the denominator 5, it makes $\frac{720}{5}$ of a penny, equal to $\frac{3}{5}$ l.

Queſt. 4. What part of an ounce Troy is $\frac{5}{16}$ lb.? *Anſ.* $\frac{60}{16}$ oz.

C H A P. XX.

Addition of Vulgar Fractions.

1. IF your fractions to be added have a common denominator, then add all the numerators together, and place their ſum for a numerator to the common denominator, which new fraction is the ſum of all the given fractions; and if it be improper, reduce it to a whole or mixed number, by the 3d rule of the 19th chapter.

Queſt. 1. What is the ſum of $\frac{7}{24}$, $\frac{9}{24}$, $\frac{16}{24}$, and $\frac{14}{24}$?

The denominators are equal, *viz.* every one is 24; wherefore add the numerators together, *viz.* 7, 9, 16, and 14, their ſum is 46; which put over the denominator 24, it makes $\frac{46}{24}$, the ſum of the given fractions; which will be reduced to the mixed number $1\frac{22}{24}$, or $1\frac{11}{12}$.

2. But if the fractions to be added have unequal denominators, then reduce them to a common denominator by the 7th rule of the 19th chapter, and then add the numerators together, and put the ſum over the common denominator, &c. as before in the laſt example.

Queſt. 2. What is the ſum of $\frac{3}{5}$, $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$?

The fractions reduced to a common denominator are $\frac{2880}{4800}$, $\frac{4200}{4800}$, $\frac{4320}{4800}$, and $\frac{4400}{4800}$; the ſum of their numerators is 15800; which put over the common denominator, makes $\frac{15800}{4800}$, or $\frac{158}{48}$, equal to the mixed number $3\frac{14}{48}$, or $3\frac{7}{24}$, for the ſum required.

Queſt. 3. What is the ſum of $\frac{13}{17}$, $\frac{21}{49}$, and $\frac{39}{47}$? *Anſ.* $2\frac{903}{39151}$.

3. If you are to add mixed numbers together, then add the fractional parts as before; and if their ſum be an improper fraction, reduce it to a mixed number, and add its integral part to the integral parts of the given mixed numbers, and the work is done.

Queſt. 4. What is the ſum of $13\frac{3}{4}$ and $24\frac{5}{8}$?

Firſt, add the fractions $\frac{3}{4}$ and $\frac{5}{8}$, the ſum is $1\frac{12}{32}$; then add the integer 1 to 13 and 24, their ſum is 38; and put after it the fraction $\frac{12}{32}$, it is $38\frac{12}{32}$, for the anſwer, or it is $38\frac{3}{8}$.

Queſt. 5. What is the ſum of $48\frac{3}{7}$, $64\frac{5}{8}$, and $130\frac{3}{4}$? *Facit* $243\frac{180}{224}$, or $243\frac{45}{56}$.

4. If any of the fractions to be added is a compound fraction, it muſt firſt be reduced to a ſimple fraction by the 6th rule of chapter 19. and then add it to the reſt according to the 2d rule of this chapter. For example:

Queſt. 6. What is the ſum of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{5}{6}$?

Reduce $\frac{7}{8}$ of $\frac{3}{4}$ of $\frac{5}{6}$ into a ſimple fraction, and it is $\frac{105}{192}$; which reduced with the other two, and added, are $2\frac{600}{4608}$.

Queſt. 7. What is the ſum of $\frac{11}{12}$, and $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$? *Anſ.* $1\frac{5}{12}$.

5. If the fractions to be added are not of one denomination, they muſt be ſo reduced, and then proceed as before.

Queſt. 8. What is the ſum of $\frac{3}{4}$ l. and $\frac{5}{6}$ s.?

Of the given fractions here, one is of a pound, and the other the fraction of a ſhilling; and before you can add them together, you muſt reduce $\frac{5}{6}$ s. to the fraction of a pound, as the other is, (by the 8th rule of chapter 19.), and it makes $\frac{5}{120}$ l.; then $\frac{3}{4}$ l. and $\frac{5}{120}$ l. will be found to be $\frac{380}{480}$ l. or $\frac{38}{48}$ l. by the 7th rule of chapter 19. and in its loweſt terms $\frac{19}{24}$ l. by the 4th rule of chapter 19.

It would have been the ſame, if (by the latter part of the 8th rule of chapter 19.) you had reduced $\frac{3}{4}$ l. to the fraction of a ſhilling; which you would have found to have been $\frac{60}{4}$ s.; which added to $\frac{5}{6}$ s. by the ſaid 7th rule of the laſt chapter, the ſum is $15\frac{20}{24}$ s. which is equal to the ſum found, as before, *viz.* $\frac{19}{24}$ l.: for (by the 5th rule of chapter 19.) the value of $\frac{19}{24}$ l. will be found to be 15 s. 10 d.; and ſo will $15\frac{20}{24}$ s. be found to be juſt as much.

Queſt. 9. What is the ſum of $\frac{3}{4}$ l. $\frac{3}{4}$ s. and $\frac{3}{4}$ d.? *Anſ.* $\frac{242880}{307200}$ l. or $\frac{24288}{30720}$ l. or in its loweſt terms $\frac{253}{320}$ l.

C H A P. XXI.

Subtraction of Vulgar Fractions.

1. THE rules in addition for reducing the given fractions to one denomination, are here to be obſerved: for before ſubtraction can be made, the fractions muſt be reduced to a common denominator; then ſubtract one numerator from the other, and place the re-

mainder over the common denominator; which fraction ſhall be the exceſs or difference between the given fractions. For example:

Queſt. 1. What is the difference between $\frac{3}{4}$ and $\frac{5}{7}$? The given fractions are reduced to $\frac{21}{28}$ and $\frac{20}{28}$; then ſubtract the numerator 20 from the numerator 21, and there remains 1; which being put over the denominator 28, makes $\frac{1}{28}$ for the anſwer, or difference between $\frac{3}{4}$ and $\frac{5}{7}$.

Queſt. 2. What is the difference between $\frac{5}{6}$ and $\frac{3}{5}$ of $\frac{5}{6}$? Reduce the compound fraction $\frac{3}{5}$ of $\frac{5}{6}$ to a ſimple fraction; then proceed as before; and the anſwer is $\frac{200}{240}$, equal to $1\frac{5}{24}$. Ans. $\frac{11}{24}$

2. When a fraction is given to be ſubtracted from a whole number, ſubtract the numerator from the denominator, and put the remainder for a numerator to the given denominator, and ſubtract an unit (for that you borrowed) from the whole number, and the remainder place before the fraction found, as before; which mixed number is the remainder or difference ſought. For example:

Queſt. 3. Subtract $\frac{7}{10}$ from 48.

Anſ. $47\frac{3}{10}$: for if you ſubtract 7 (the numerator) from 10 (the denominator), there remains 3; which put over 10 is $\frac{3}{10}$, and 1 I borrowed from 48, reſts 47, to which join $\frac{3}{10}$, and it makes $47\frac{3}{10}$ for the exceſs.

Queſt. 4. Subtract $\frac{13}{21}$ from 57, remains $56\frac{8}{21}$.

3. If it be required to ſubtract a fraction from a mixed number, or one mixed number from another, reduce the fraction to a common denominator; and if the fraction to be ſubtracted be leſs than the other, then ſubtract the leſſer numerator from the greater, and that is a numerator for the common denominator. Then ſubtract the leſſer integral part from the greater, and the remainder with the remaining fractions thereto annexed is the difference required between the two given mixed numbers. For example:

Queſt. 5. Subtract $26\frac{3}{7}$ from $54\frac{5}{6}$.

Firſt, ſubtract $\frac{3}{7}$, *viz.* $\frac{18}{42}$ from $\frac{5}{6}$, *viz.* $\frac{35}{42}$ the remainder is $\frac{17}{42}$; then 26 from 54, remaineth 28; to which annex $\frac{17}{42}$, it makes $28\frac{17}{42}$ for the anſwer.

4. But if the fraction to be ſubtracted is greater than the fraction from whence you ſubtract, then having firſt reduced the fractions to a common denominator, take the

numerator of the greateſt fraction out of the denominator, and add the remainder to the numerator of the leſſer fraction, and their ſum is a new numerator to the common denominator, which fraction note; then (for the 1 you borrowed) add 1 to the integral part to be ſubtracted, and ſubtract it from the greater number, and to the remainder annex the fraction you noted before; ſo this new mixed number ſhall be the difference ſought. For example:

Queſt. 6. Subtract $14\frac{3}{4}$ from $29\frac{4}{7}$.

The fractions reduced are $\frac{21}{28}$ and $\frac{16}{28}$, *viz.* $\frac{3}{4}$ equal to $\frac{21}{28}$, and $\frac{4}{7}$ equal to $\frac{16}{28}$: now, I ſhould ſubtract $\frac{21}{28}$ from $\frac{16}{28}$, but I cannot; therefore I ſubtract 21 from 28, reſts 7; which added to 16 (the leſſer numerator), makes 23 for a numerator to 28, *viz.* $\frac{23}{28}$. Then I come to the integral parts 14 and 29; and ſay, 1 that I borrowed and 14 is 15, which taken from 29, there reſts 14; to which annexing $\frac{23}{28}$ it is $14\frac{23}{28}$ for the remainder, or difference between $14\frac{3}{4}$ and $29\frac{4}{7}$.

Queſt. 7. Subtract $36\frac{9}{10}$ from $74\frac{4}{9}$. *Facit* $37\frac{49}{90}$.

C H A P. XXII.

Multiplication of Vulgar Fractions.

1. IF the multiplicand and multiplier are ſimple or ſingle fractions, then multiply the numerators together for a new numerator, and the denominators for a new denominator, and the new fraction is the product required.

Queſt. 1. What is the product of $\frac{5}{7}$ by $\frac{9}{11}$? *Facit* $\frac{45}{77}$; for the numerators 5 and 9 being multiplied, make 45; and the denominators 7 and 11 being multiplied, make 77.

Queſt. 2. What is the product of $\frac{18}{25}$ by $\frac{21}{37}$? *Facit* $\frac{378}{925}$.

2. If the fractions to be multiplied be mixed numbers, reduce them to improper fractions by the 1ſt rule of the 19th chapter; then proceed as before.

Queſt. 3. What is the product of $48\frac{3}{5}$ by $13\frac{5}{6}$?

The given mixed numbers being reduced to improper fractions are $48\frac{3}{5}$ equal to $\frac{243}{5}$, and $13\frac{5}{6}$ equal to $\frac{83}{6}$: now $\frac{243}{5}$ multiplied by $\frac{83}{6}$, according to the 1ſt rule of this chapter, produceth $\frac{20169}{30}$, or $672\frac{9}{30}$.

Queſt. 4. What is the product of $430\frac{6}{10}$ by $18\frac{1}{4}$? *Facit* [illegible], or $7935\frac{24}{40}$.

3. If a compound fraction is to be multiplied by a ſimple fraction, firſt reduce the compound fraction into a ſimple fraction; then multiply the one by the other, as is taught above.

Queſt. 5. What is the product of $\frac{16}{21}$ by $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{5}$?

The compound fraction $\frac{3}{4}$ of $\frac{5}{7}$ of $\frac{4}{5}$ reduced is $\frac{60}{140}$, or $\frac{6}{14}$; which multiplied by $\frac{16}{21}$, produceth $\frac{96}{294}$; which in its loweſt terms is $\frac{16}{49}$ for the anſwer.

And if the multiplicand and multiplier are both compound fractions, reduce them both to ſimple ones: then multiply theſe new fractions as before, ſo you have the product.

Queſt. 6. What is the product of $\frac{3}{4}$ of $\frac{2}{3}$ by $\frac{3}{5}$ of $\frac{1}{2}$? *Anſ.* $\frac{18}{120}$, in its loweſt terms $\frac{3}{20}$.

Queſt. 7. What is the product of $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{2}{5}$ of $\frac{5}{6}$? *Anſ.* $\frac{60}{360}$, or $\frac{6}{36}$, or in its loweſt terms $\frac{1}{6}$.

4. If a fraction be to be multiplied by a whole number, put under the given whole number an unit for a denominator, whereby it will be an improper fraction; then multiply theſe fractions as before. For example:

Queſt. 8. What is the product of 24 by $\frac{2}{3}$? *Anſ.* $\frac{48}{3}$: for 24, by putting an unit under it, will be $\frac{24}{1}$; and $\frac{2}{3}$ multiplied by $\frac{24}{1}$, produceth $\frac{48}{3}$ or 16.

Queſt. 9. What is the product of 36 by $\frac{9}{11}$? *Anſ.* $\frac{324}{11}$, or $29\frac{5}{11}$.

CHAP. XXIII.

Diviſion of Vulgar Fractions.

1. If the dividend and the diviſor are both ſimple fractions, then multiply the numerator of the dividend into the denominator of the diviſor, and the product is a new numerator; and multiply the denominator of the dividend into the numerator of the diviſor, and the product is a new denominator; which new fraction thus found, is the quotient you deſire. For example:

Queſt. 1. What is the quotient of $\frac{5}{8}$ divided by $\frac{3}{5}$? *Anſ.* $\frac{25}{24}$, or $1\frac{1}{24}$. For firſt I multiply (5) the numerator of the divi-

$$\frac{3}{5}\Big)\frac{5}{8}\Big(\frac{25}{24}$$

dend into (5) the denominator of the diviſor, and the product (25) is a numerator for the quotient: Then I multiply (8) the denominator of the dividend into (3) the numerator of the diviſor, and the product (24) I put in the quotient for a denominator. So I find $\frac{25}{24}$ is the quotient ſought.

Queſt. 2. What is the quotient of $\frac{10}{21}$ divided by $\frac{2}{3}$? *Anſ.* $\frac{30}{42}$, equal to $\frac{5}{7}$ in its loweſt terms.

2. But if you would divide a ſimple fraction by a compound, or a compound by a ſimple, firſt reduce ſuch compound to a ſimple fraction; then go on as before.

Queſt. 3. What is the quotient of $\frac{3}{10}$ divided by $\frac{3}{4}$ of $\frac{2}{3}$? *Anſ.* $\frac{36}{60}$, or $\frac{3}{5}$. Firſt reduce $\frac{3}{4}$ of $\frac{2}{3}$ into a ſimple fraction, and it is $\frac{6}{12}$; by which $\frac{3}{10}$ being divided, the quotient is $\frac{36}{60}$, equal in its loweſt terms to $\frac{3}{5}$.

And if the dividend and diviſor be both compound fractions, reduce them both to ſimple fractions; then divide the one by the other, as in rule 1. foregoing.

Queſt. 4. What is the quote of $\frac{2}{3}$ of $\frac{3}{4}$ divided by $\frac{2}{5}$ of $\frac{5}{6}$? *Anſ.* $\frac{180}{125}$, or $\frac{18}{12}$, or $1\frac{6}{12}$, or $1\frac{1}{2}$ in its loweſt terms.

3. If the dividend, or diviſor, or both, are mixed numbers, reduce them to improper fractions, and perform diviſion as you were taught before.

Queſt. 5. What is the quote of $12\frac{3}{4}$ divided by $21\frac{4}{5}$? *Anſ.* $\frac{255}{436}$: for $12\frac{3}{4}$ is equal to $\frac{51}{4}$, and $21\frac{4}{5}$ is equal to $\frac{109}{5}$; and the quote of $\frac{51}{4}$, divided by $\frac{109}{5}$, is, as before, $\frac{255}{436}$.

4. If you divide a fraction by a whole number, or a whole number by a fraction, make the whole number an improper fraction, by putting an unit for a denominator to it, as was taught in rule 4. of chapter 22. and then perform diviſion as was before taught. For example:

Queſt. 6. What is the quote of 8 divided by $\frac{3}{5}$? *Anſ.* $\frac{40}{3}$, which is equal to $13\frac{1}{3}$, being reduced as is before directed. See the work in the margin.

$$\frac{3}{5}\Big)\frac{8}{1}\Big(\frac{40}{3} \text{ or } 13\tfrac{1}{3}$$

Queſt. 7. What is the quotient of $\frac{3}{5}$ divided by 8? *Anſ.* $\frac{3}{40}$, as *per* margin.

$$\frac{8}{1}\Big)\frac{3}{5}\Big(\frac{3}{40}$$

CHAP. XXIV.

The Rule of Three direct in Vulgar Fractions.

1. AS in the rule of three in whole numbers, ſo likewiſe in fractions, you muſt ſee that the fractions of the firſt and third places be of the ſame denomination.

2. See, if any of the given fractions be compound, that they be reduced to ſimple ones of the ſame value.

3. If there are given mixed numbers, reduce them to improper fractions by the 1ſt rule of chapter 19.

4. If any of the three terms is a whole number, make an improper fraction, by conſtituting an unit for its denominator.

Having reduced your fractions as is directed in the four firſt rules, then proceed to a reſolution, which is performed the ſame way as in whole numbers, reſpect being had to the rules delivered for the working of fractions, *viz* Multiply the ſecond and third fractions together, according to the rules of chapter 22. and divide the product by the firſt fraction, according to the rules of chapter 23. and the quotient is the anſwer. Or, which is better,

5. Multiply the numerator of the firſt fraction into the denominators of the ſecond and third, and the product is a new denominator; then multiply the denominator of the firſt fraction into the numerators of the ſecond and third, and the product is a new numerator; which new fraction is the fourth proportional or anſwer; which, if it be an improper fraction, muſt be reduced to a whole or mixed number by the 3d rule of chapter 19. For example:

Queſt. 1. If $\frac{3}{4}$ yard of cloth coſt $\frac{5}{8}$ l. what will $\frac{9}{10}$ yard coſt?

Having placed the given fractions according to the 6th rule of chapter 10. I proceed to the reſolution. And firſt I multiply the numerator of the firſt fraction (3) into 8 and 10, the denominators of the ſecond and third fractions, and the product is 240 for a denominator; then I

Yards. *l.* *Yards* . *l.*

$\frac{3}{4} : \frac{5}{8} :: \frac{9}{10} : \frac{180}{240}$

Facit $\frac{180}{240}$, equal to $\frac{3}{4}$ l. or 15 s.

multiply 4 the denominator of the firſt fraction into 5 and 9, the numerators of the ſecond and third fractions, the product is 180 for a numerator; which numerator 180, and denominator 240, make $\frac{180}{240}$ l. for the anſwer, equal to $\frac{3}{4}$ l. or 15 s.

Queſt. 2. If $\frac{2}{3}$ l. buy $\frac{5}{6}$ yard of cloth, what will $\frac{11}{12}$ yard coſt at that rate? *Anſ.* $\frac{132}{180}$ l. equal to $\frac{11}{15}$ l. or 14 s. 8 d.

Queſt. 3. If $\frac{7}{8}$ lb. coſt $\frac{3}{4}$ s. what will $\frac{8}{9}$ s. buy? *Anſ.* $\frac{224}{216}$ lb. equal to $1\frac{1}{27}$ lb.

Queſt. 4. If $\frac{3}{5}$ of an ell of holland coſt $\frac{1}{3}$ of a pound, how much will $12\frac{2}{3}$ ells coſt at that rate? *Anſ.* $\frac{190}{27}$ l. equal to $7\frac{1}{27}$ l.

In reſolving the laſt queſtion and the two next, obſerve the third rule of this chapter.

Queſt. 5. If $\frac{9}{10}$ C. coſt $28\frac{3}{4}$ s. what will $7\frac{1}{2}$ C. coſt at that rate? *Anſ.* $239\frac{7}{12}$ s. or 11 l. 19 s. 7 d.

Queſt. 6. If $3\frac{1}{4}$ yards of velvet coſt $3\frac{5}{8}$ l. how much will $10\frac{1}{2}$ yards coſt at that rate? *Anſ.* $11\frac{37}{52}$ l.

Queſt. 7. If 3 yards of broad-cloth coſt $2\frac{4}{5}$ l. what will $14\frac{1}{7}$ yards coſt? *Anſ.* 13 l. 9 s. 4 d.

In working the laſt queſtion and the four next, obſerve the 4th rule of this chapter.

Queſt. 8. If 14 lb. of pepper coſt 14 s. $6\frac{3}{5}$ d. I demand the price of $73\frac{3}{4}$ lb.? *Anſ.* 3 l. 16 s. $7\frac{43}{56}$ d.

Queſt. 9. If 1 lb. of cochineal coſt 1 l. 5 s. what will $36\frac{7}{10}$ lb. coſt? *Anſ.* 45 l. 17 s. 6 d.

Queſt. 10. If 1 yard of broad-cloth coſt $15\frac{5}{8}$ s. what will 4 pieces, each containing $27\frac{3}{7}$ yards, coſt at that rate? *Anſ.* 85 l. 14 s. $3\frac{3}{7}$ d.

Queſt. 11. A mercer bought $3\frac{1}{2}$ pieces of ſilk, each piece containing $24\frac{2}{3}$ ells at 6 s. $\frac{3}{4}$ d. *per* ell; I demand the value of $3\frac{1}{2}$ pieces at that rate? *Anſ.* 26 l. 3 s. $4\frac{3}{4}$ d.

In reſolving the four next queſtions, obſerve the 8th rule of chapter 19.

Queſt. 12. If $\frac{2}{5}$ of an ounce of ſilver coſt 2 s. I demand the price of $11\frac{2}{3}$ lb. at that rate? *Anſ.* 35 l.

Queſt. 13. If $1\frac{5}{7}$ lb. of gold is worth $61\frac{5}{7}$ l. Sterling, what is a grain worth at that rate? *Anſ.* $1\frac{1}{2}$ d.

Queſt. 14. If $\frac{3}{4}$ yard of ſilk is worth $\frac{3}{4}$ of $\frac{5}{6}$ l. what is the price of $15\frac{2}{5}$ ells Flemiſh? *Anſ.* 9 l. 12 s. 6 d.

Queſt. 15. If $\frac{2}{7}$ of $\frac{3}{4}$ of a pound of cloves coſt 6 s. $2\frac{2}{7}$ d. what coſt the C. weight at that rate? *Anſ.* 69 l. 6 s. 8 d.

Note, That when the anſwers to the queſtions in this and the next chapter are given in fractions, they are given in their loweſt terms.

CHAP. XXV.

The Rule of Three inverſe in Fractions.

1. IT hath been already taught (in the 3d rule of the 11th chapter) how to diſcover when the fourth proportional number (to the three given numbers) is to be found out by the rule of three direct, and when by the rule of three inverſe; to which rule the learner is now referred.

2. When in fractions you find a queſtion to be ſolved by the rule of three inverſe, *viz.* when the third term is the diviſor; then having reduced the terms exactly, (according to the rules in chapter 24.), multiply the numerator of the third fraction into the denominators of the ſecond and firſt fractions, and the product is a new denominator; then multiply the denominator of the third fraction into the numerators of the ſecond and firſt fractions, and the product is a new numerator; which new fraction thus found is the anſwer to the queſtion.

Queſt. 1. If $\frac{3}{4}$ of a yard of cloth that is 2 yards wide will make a garment, how much of any other drapery that is $\frac{3}{5}$ of a yard wide will make the ſame garment? *Anſ.* $2\frac{1}{2}$ yards.

Queſt. 2. I lent my friend 46 l. for $\frac{4}{5}$ of a year, how much ought he to lend me for $\frac{7}{12}$ of a year? *Anſ.* $63\frac{3}{35}$ l.

Queſt. 3. If $\frac{2}{3}$ of a yard of cloth that is $2\frac{1}{3}$ yards wide will make any garment, what breadth is that cloth when $1\frac{3}{4}$ yards will make the ſame garment? *Anſ.* $\frac{56}{63}$ of a yard wide.

Queſt. 4. How many inches in length of a board that is 9 inches broad will make a foot ſquare? *Anſ.* 16 inches in length.

Queſt 5. If when the buſhel of wheat coſt $4\frac{1}{4}$ s. the penny loaf weighed $10\frac{2}{3}$ ounces, what will it weigh when the buſhel coſt $8\frac{2}{15}$ s.? *Anſ.* $5\frac{185}{267}$ ounces.

Queſt. 6. If 12 men can mow $24\frac{1}{2}$ acres in $10\frac{2}{3}$ days, in how many days will 6 men do the ſame? *Anſ.* in $21\frac{1}{3}$ days.

CHAP. XXVI.

Rules of Practice.

1. IN the ſingle rule of three, when the firſt of the three numbers in the queſtion (after they are diſpoſed according to the 6th rule of chapter 10.) happeneth to be an unit, or 1, that queſtion many times may be reſolved far more ſpeedily than by the rule of three; which kind of operation is commonly called *Practice*. And indeed it is of excellent uſe among merchants, tradeſmen, and others, by reaſon of its ſpeedineſs in finding a reſolution to ſuch kind of queſtions.

2. The chief queſtions reſolvable by theſe brief rules may be comprehended under the ſeven general heads or caſes following, *viz.*

When the given price of the integer conſiſts,
1. Of farthings under 4.
2. Of pence under 12.
3. Of pence and farthings.
4. Of ſhillings under 20.
5. Of ſhillings, pence, and farthings.
6. Of pounds.
7. Of pounds, ſhillings, pence, and farthings.

It would be very convenient for the practical arithmetician to have by heart the ſeveral products of the 9 digits multiplied by 12, for his ſpeedy reducing pence into ſhillings, and ſhillings into pence, which he may gain by the table on the margin.

12 times	1	is	12
	2		24
	3		36
	4		48
	5		60
	6		72
	7		84
	8		96
	9		108

3. Shillings are practically reduced into pounds thus, *viz.* Cut off the figure ſtanding in the place of units with a daſh of the pen, and note it for ſhillings; then draw a line under the given number, and take half of the remaining figures after the firſt is cut off, and ſet them under the line, and they are ſo many pounds: but if the laſt figure is odd, then take the leſſer half, and add 10 to the figure ſo cut off (as before) for ſhillings. As, if I were to reduce 43658 ſhillings into pounds; firſt I cut off the laſt figure (8) for ſhillings, then I take half of the remaining figures (4365) thus. Half of 4 is 2, which I put under the line; then half of three is 1; and becauſe 3 is an odd number, I make the next figure 6 to be 16; and I go on, ſaying, Half of 16 is 8, and then half of 5 is 2, which is the laſt figure; wherefore becauſe 5 is an odd number, I add 10 to the 8 I cut off, and it makes 18 s. So that I find it to be 2182 l. 18 s. as *per* margin.

4365|8

l.	s.
2182	18

4. It is likewiſe convenient that the learner be acquainted with the practical tables following; the firſt containing the aliquot or even parts of a ſhilling, the ſecond containing the aliquot parts of a pound.

	d.		s.
The even parts of a ſhilling.	6	is	$\frac{1}{2}$
	4		$\frac{1}{3}$
	3		$\frac{1}{4}$
	2		$\frac{1}{6}$
	1$\frac{1}{2}$		$\frac{1}{8}$
	1		$\frac{1}{12}$

	s.	d.		l.
The even parts of a pound.	10	0	is	$\frac{1}{2}$
	6	8		$\frac{1}{3}$
	5	0		$\frac{1}{4}$
	4	0		$\frac{1}{5}$
	3	4		$\frac{1}{6}$
	2	6		$\frac{1}{8}$
	2	0		$\frac{1}{10}$
	1	8		$\frac{1}{12}$
	1	0		$\frac{1}{20}$

5. *Caſe* 1. When the price of an integer is a farthing, then take the ſixth part of the given number, which will be ſo many three-halfpences; and if any thing remains, it is farthings by the 8th rule of chapter 8. Then conſider that three-halfpences is $\frac{1}{8}$ of a ſhilling, wherefore take the 8th part of them for ſhillings, and if any thing remain, they are ſo many three-halfpences: which reduce into pounds

by the 3d rule foregoing. *Example*, What comes 67486 lb. to, at a farthing *per* lb.? Firſt, I take $\frac{1}{6}$ of 67486, and it is 11247 three-halfpences, and 4 farthings, or 1 penny. Then $\frac{1}{8}$ of 11247 is 1405 s. and 7 remains, which is 7 three-halfpences, or 10½ d.; which, with the 4 farthings before, make 11½ d.; and 1405 ſhillings, by the 3d rule, is 70 l. 5 s.: in all 70 l. 5 s. 11½ d. for the anſwer. See the work.

	lb. *d.*
$\frac{1}{6}$	67486 at ¼ *per* lb.
$\frac{1}{8}$	11247—1
$\frac{1}{20}$	140|5 —11½
	l. *s.* *d.*
	70—5—11½ *facit.*

Other examples follow.

$\frac{1}{6}$	8576 lb. at 1 qr.	$\frac{1}{6}$	6380 lb. at 1 qr.
$\frac{1}{8}$	1429——2 qrs.	$\frac{1}{8}$	1063——2 qrs.
$\frac{1}{20}$	17|8——8 d.	$\frac{1}{20}$	13|2——11 d.
	l. *s.* *d*		*l.* *s* *d.*
	8—18—8 *facit.*		6—12—11 *facit.*

6. When the price of the integer is 2 farthings, then take the third part of the given number for ſo many three-halfpences, and the remainder (if any) is halfpence; then take the eighth part of that for ſhillings, as before, *&c.*

Examples.

$\frac{1}{3}$	7368 lb. at 2 qrs.	$\frac{1}{3}$	8347 lb. at 2 qrs.
$\frac{1}{8}$	2456	$\frac{1}{8}$	2782——2 qrs.
$\frac{1}{20}$	30|7	$\frac{1}{20}$	34|7——9½ d.
	l. *s.*		*l.* *s.* *d.*
	15—7 *facit.*		17—7—9½ *facit.*

7. When the price of the integer is 3 farthings, then take half the given number for three-halfpence, and if any thing remain, it is three farthings; then take the eighth of that for ſhillings, as before, *&c.*

$\frac{1}{2}$	4736 lb. at 3 qrs.
$\frac{1}{8}$	2368
$\frac{1}{20}$	29|6
	l. *s.*
	14—16 *facit.*

$\frac{1}{2}$	5425 lb. at 3 qrs.
$\frac{1}{8}$	2712——3 qrs.
$\frac{1}{20}$	33|9
	l. *s.* *d.* *qrs.*
	16—19—0—3 *facit.*

8. *Caſe* 2. When the given price of the integer is a part or parts of a ſhilling, (*viz.* pence), divide the given number of integers whoſe value is ſought, by the denominator of the fraction repreſenting the even part, and the quote is ſhillings; (always minding the 8th rule of the 8th chapter, for the value of the remainder); and thoſe ſhillings may be reduced into pounds by the 3d rule of this chapter. For example: Let it be required to find the value of 438 l. at 3 d. *per* lb. I conſider that 3 d. is $\frac{1}{4}$ of a ſhilling, and 438 lb. will coſt ſo many three-pences; wherefore I divide 438 by 4, the denominator of $\frac{1}{4}$, and the quote is 109 ſhillings, and 2 remains, which is 2 three-pences, or 6 d.: the whole value is 5 l. 9 s. 6 d. as by the work appeareth.

$\frac{1}{4}$	438 lb at 3 d.
$\frac{1}{20}$	10|9 —— 6
	l. *s.* *d.*
	Facit 5—9—6

More examples follow.

	lb. *d.*
$\frac{1}{2}$	3574 at 6 *per* lb.
$\frac{1}{20}$	178|7
	89 l. 7 s. *facit.*

	lb. *d.*
$\frac{1}{6}$	5316 at 2 *per* lb.
$\frac{1}{20}$	88|6
	44 l. 6 s. *facit.*

	lb. *d.*
$\frac{1}{3}$	438 at 4 *per* lb.
$\frac{1}{20}$	14|6
	7 l. 6 s. *facit.*

	lb. *d.*
$\frac{1}{8}$	6389 at 1$\frac{1}{2}$ *per* lb.
$\frac{1}{20}$	79|8—7$\frac{1}{2}$ d.
	39 l. 18 s. 7$\frac{1}{2}$ d. *fac.*

	lb. *d.*
$\frac{1}{4}$	879 at 3 *per* lb.
$\frac{1}{20}$	21|9—9
	10 l. 19 s. 9 d. *facit.*

	lb. *d.*
$\frac{1}{12}$	818 at 1 *per* lb.
$\frac{1}{20}$	6|8——2 d.
	3 l. 8 s. 2 d. *facit.*

9.

9. If the price of the integer be pence under 12, and yet not an even part, then it may be divided into even parts; and ſo the parts of the given number taken accordingly, and added together, as if it were 5 d. which is 3 d. and 2 d. *viz.* $\frac{1}{4}$ and $\frac{1}{6}$ of a ſhilling; firſt take $\frac{1}{4}$ of the given number, and then $\frac{1}{5}$ thereof, and add them together, and their ſum is the anſwer in ſhillings; ſtill obſerving rule 8. of chap. 8. for the remainder, (if any be); then bring the ſhillings into pounds by the 3d rule foregoing. Likewiſe 7 d. is $\frac{1}{3}$ and $\frac{1}{4}$, ſo 9 d. is $\frac{1}{2}$ and $\frac{1}{4}$, and 10 d. is $\frac{1}{2}$ and $\frac{1}{3}$, and 11 d. is $\frac{1}{3}$ and $\frac{1}{3}$ and $\frac{1}{4}$ of a ſhilling. Or elſe many times your work may be ſhortened thus, *viz.* When the ſaid given price is to be divided into even parts of a ſhilling or of a pound; after you have taken the firſt even part, the other may be an even part of that part; as in the next example, where is given 439 lb. at 5 d. *per* lb. Now, I may divide it thus, *viz.* into 4 d. 1 d.; and 4 d. being $\frac{1}{3}$ of a ſhilling, and 1 d. being $\frac{1}{4}$ of 4 d. I firſt take $\frac{1}{3}$ of 439 lb. and it gives 146 s. 4 d.; and for the 1 d. I take $\frac{1}{4}$ of 146 s. 4 d. which is 36 s. 7 d; which in all comes to 9 l. 2 s. 11 d. Examples follow.

	lb. *d.*
$\frac{1}{3}$	439 at 5 *per* lb.
$\frac{1}{4}$	146——4
	36——7
$\frac{1}{20}$	18|2—11
	9 l. 2 s. 11 d. *facit.*

	Yards. *d.*
$\frac{1}{2}$	417 at 9 *per* yard.
$\frac{1}{2}$	208——6
	104——3
$\frac{1}{20}$	31|2——9
	15 l. 12 s. 9 d. *facit.*

	Ells. *d.*
	587 at 7 *per* ell.
$\frac{1}{3}$	195——8
$\frac{1}{4}$	146——9
$\frac{1}{20}$	34|2——5
	17 l. 2 s. 5 d. *facit.*

	Ells. *d.*
	386 at 10
$\frac{1}{2}$	193
$\frac{1}{3}$	128——8
$\frac{1}{20}$	32|1——8
	16 l. 1 s. 8 d. *facit.*

Yards.

	Yards. *d.*
	836 at 8 *per* yard.
$\frac{1}{3}$	278——8
$\frac{1}{3}$	278——8
$\frac{1}{20}$	55|7——4
	27 l. 17 s. 4 d. *facit.*

	lb. *d.*
	534 at 11
$\frac{1}{3}$	178
$\frac{1}{3}$	178
$\frac{1}{4}$	133——6 d.
$\frac{1}{20}$	48|9——6
	24 l. 9 s. 6 d. *facit.*

10. *Caſe* 3. When the price of the integer is pence and farthings, if it make an even part of a ſhilling, work as before: but if they are uneven, as penny-farthing, penny three-farthings, 2 d. 1 qr. or 2 d. 3 qrs. 3 d. 3 qrs. or the like; then firſt work for ſome even part, and then conſider what part the reſt is of that even part, and divide that quotient thereby; then add them together, and reduce them to pounds, as before

Example. 3470 lb. at 1 d. 1 qr. *per* lb. Firſt, I work for the penny, by dividing 3470 l. by 12, for 1 d. is $\frac{1}{12}$ of a ſhilling, and the quote is 289 s. 2 d. Then I conceive that one farthing is the $\frac{1}{4}$ of a penny, and the value of 1 farthing will be $\frac{1}{4}$ of the value of a penny; and therefore I take $\frac{1}{4}$ of 289 s. 2 d. which is 72 s. 3 d. 2 qrs. and add them together, and they are 18 l. 1 s. 5 d. 2 qrs. as by the margin. Other examples of the ſame nature follow.

	lb. *d.* *qr.*
$\frac{1}{12}$.	3470 at 1—1
$\frac{1}{4}$	289——2
	72——3—$\frac{1}{2}$
$\frac{1}{20}$	36|1——5$\frac{1}{2}$
	l. *s.* *d.* *qrs.*
	18—1—5—2

	lb. *d.*
$\frac{1}{12}$	4360 at 1$\frac{1}{4}$
$\frac{1}{4}$	363——4
	90——10
$\frac{1}{20}$	45|4——2
	l. *s.* *d.*
	22—14—2 *facit.*

	Yards. *d.*
$\frac{1}{12}$	859 at 1$\frac{1}{8}$
$\frac{1}{8}$	71——7 qr.
	8——11——1$\frac{1}{2}$
$\frac{1}{20}$	8|0——6——1$\frac{1}{2}$
	l. *s.* *d.* *qr.*
	4—0—6—1$\frac{1}{2}$ *facit.*

lb.

	lb. *d.*
$\frac{1}{6}$	485 at $2\frac{1}{4}$
$\frac{1}{8}$	80—10
	10—$1\frac{1}{4}$
$\frac{1}{20}$	9|0—$11\frac{1}{4}$
	4 l. 10 s. $11\frac{1}{4}$ d. *facit.*

	Yards. *d.*
$\frac{1}{2}$	520 at $7\frac{1}{2}$
$\frac{1}{4}$	260
	65
$\frac{1}{20}$	32|5
	16 l. 5 s. *facit.*

$\frac{1}{6}$	654 lb. at $2\frac{1}{4}$ d.
$\frac{1}{4}$	109
	27—3
$\frac{1}{20}$	13|6—3
	6 l. 16 s. 3 d. *facit.*

$\frac{1}{2}$	137 yds at $10\frac{1}{2}$ d.
$\frac{1}{2}$	68—6
$\frac{1}{2}$	34—3
	17—$1\frac{1}{2}$
$\frac{1}{20}$	11|9—$10\frac{1}{2}$
	5 l. 19 s. $10\frac{1}{2}$ d. *fac.*

11. *Caſe* 4. When the price of the integer is 2 s. then cut off the figure in the place of units of the given number, and double it for ſhillings, and the figures on the other hand are pounds. *Example:* 436 yards at 2 s. *per* yard; cut off the laſt figure 6, and double it, it makes 12 ſhillings; and the other two figures, *viz.* 43, are ſo many pounds: ſo that their value is 43 l. 12 s. as *per* margin.

43|6

43 l. 12 s.

12. Hence it is evident, that when the given price of an integer is an even number of ſhillings, then if you take half of that (even) number of ſhillings, and multiply the given number of integers thereby, doubling the firſt figure of the product, and ſetting it apart for ſhillings, the reſt of the product will be pounds; which pounds and ſhillings are the value ſought. *Example.* What coſt 536 yards at 8 s *per* yard? To reſolve which, I take $\frac{1}{2}$ of 8 s. (the price of a yard), which is 4, and multiply 536 thereby; ſaying, 4 times 6 is 24; then I double the firſt figure 4, which makes 8, for ſhillings, and carry 2 to the next product, *&c.* and I find the reſt of the product to be 214, which I note for pounds: ſo the value of 536 yards at 8 s. *per* yard, is 214 l. 8 s. as *per* margin. More examples follow.

536 yds at 8 s.

214 l. 8 s.

56 yds at 6 s. *per* yd.

16 l. 16 s. *facit.*

123 yds at 4 s. *per* yd.

24 l. 12 s. *facit.*

48 ells at 8 s. *per* ell.

19 l. 4 s. *facit.*

84 yds at 10 s. *per* yd.

42 l. *facit.*

420 yds at 12 s. *per* yd.

252 l. *facit.*

326 yds at 14 s. *per* yd.

228 l. 4 s. *facit.*

48 yds at 16 s. *per* yd.

38 l. 8 s. *facit.*

52 yds at 18 s. *per* yd.

46 l. 16 s. *facit.*

13. If the given price of the integer is an odd number of ſhillings, then work firſt for the even number of ſhillings, by the laſt rule; and for the odd ſhilling take $\frac{1}{20}$ of the given number of integers, according to the 3d rule of this chapter, and add them together, and you have your deſire. Examples follow.

Yds. *s.*
422 at 3 *per* yd.

l. *s.*
42——4
21——2

63——6 *facit.*

Ells *s.*
516 at 7 *per* ell.

l. *s.*
154——16
25——16

180——12 *facit.*

Ells. *s.*
431 at 13

l. *s.*
258——12
21——11

280—— 3 *facit.*

Ells. *s.*
324 at 17 *per* ell.

l. *s.*
259——4
16——4

275——8 *facit.*

14. Except when the given price of the integer is 5 s.; for then it is ſooner anſwered by taking ¼ of the given number whoſe value is ſought, as in the following examples.

¼ | *Yds.* *s.*
436 at 5 *per* yd.
109 l. *facit.*

¼ | *Ells.* *s.*
206 at 5 *per* ell.
51 l. 10 s. *facit.*

15. *Caſe* 5. When the given price of an integer is ſhillings and pence, or ſhillings, pence, and farthings; then if the ſhillings and pence be an even part of a pound, divide the given number of integers whoſe value you ſeek, by the denominator of that fraction repreſenting that even part. As for example: What is the price of 384 yards at 6 s. 8 d. *per* yard? Here I conſider that 6 s. 8 d. is $\frac{1}{3}$ of a pound; wherefore I divide 384 by 3, and the quote is the anſwer, *viz.* 128 l.; ſo that 384 yards at 6 s. 8 d. *per* yard, amount to 128 l. as *per* margin, ſtill obſerving the 8th rule of the 8th chapter.

	Yds. s. d.
$\frac{1}{3}$	384 at 6—8 *per* yd.
	128 l. *facit.*

More examples follow.

$\frac{1}{3}$	438 ells at 6 s. 8 d.	$\frac{1}{8}$	443 yds at 2 s. 6 d.
	146 l. *facit.*		55 l. 7 s. 6 d. *facit.*
$\frac{1}{6}$	1125 ells at 3 s. 4 d.	$\frac{1}{12}$	726 yds at 1 s. 8 d.
	187 l. 10 s. *facit.*		60 l. 10 s. *facit.*

16. When the given value of the integer is ſhillings and pence, *&c.* and not an even part of a pound, yet many times it may be divided into even parts. Thus 6 s. 6 d. is 4 s. and 2 s. 6 d. For the 4 s. work according to the 12th rule foregoing, and for the 2 s. 6 d. take the eighth part of the given number, and add them together, then their ſum is the value required.

So 8 s. 6 d. will be divided into 6 s. and 2 s. 6 d. and the price of the given number may be found out as before, *&c.* Examples follow.

	Yds. s. d.		*Ells.* s. d.
	386 at 8—8		540 at 5—4
$\frac{1}{3}$	128 l.—13—4	2 s	54——0 s.
2s.	38 —12—0	$\frac{1}{6}$	90——0 s.
	167 l. 5 s. 4 d. *facit.*		144 l. *facit.*

Ells

	Ells. s. d.
	427 at 8—6
6 s	128 l.—2—0
$\frac{1}{8}$	53 —7—6
	181 l. 9 s. 6 d. *facit.*

	Yds. s. d.
	386 at 14—8
8 s	154 l.— 8—0
$\frac{1}{3}$	128 —13—4
	283 l. — 1—4 *facit.*

17. When the given price of an integer is ſhillings and pence, and you cannot readily divide them according to the laſt rule, then multiply the given number whoſe value you ſeek, by the number of ſhillings in the price of the integer; and then for the pence work by the 8th rule foregoing; then add the numbers together, and their ſum is the value ſought in ſhillings. As for example: What is the value of 392 yards at 6 s. 9 d. *per* yard? Here 6 s. 9 d. cannot be made an even part; nor indeed can it be divided into even parts of a pound: wherefore I multiply the given number of yards, 392 by 6, for the 6 s. the product is 2352 ſhillings; then for the 9 d. I divide it into 6 d. and 3 d. and work for them by the 8th rule foregoing, and at laſt add the ſhillings together, they make 2646 s.; and by the 3d rule they are reduced to 132 l. 6 s. the value of 392 yards at 6 s. 9 d. *per* yard. See the work in the margin.

	Yds. s. d.
	392 at 6—9
6 s	2352
$\frac{1}{2}$	196
$\frac{1}{4}$	98
$\frac{1}{20}$	264|6
	132 l. 6 s. *facit.*

Other examples follow.

	lb. s. d.
	480 at 4—10
4 s	1920
$\frac{1}{2}$	240
$\frac{1}{3}$	160
$\frac{1}{20}$	232|0
	116 l. *facit.*

	Ells. s. d.
	732 at 12—7
	12
12 s	8784
$\frac{1}{3}$	244
$\frac{1}{4}$	183
$\frac{1}{20}$	921|1
	460 l. 11 s. *facit.*

18. When the given price of the integer is ſhillings, pence, and farthings, then multiply the given number of integers by the number of ſhillings contained in the value of the integer; and for the pence and farthings follow the 10th rule of this chapter.

Examples.

	Yds. *s.* *d.*
	438 at 8—$6\frac{3}{4}$
8 s	3504
$\frac{1}{2}$	219
$\frac{1}{8}$	27—$4\frac{1}{2}$ *d.*
$\frac{1}{20}$	375|0—$4\frac{1}{2}$
	187 l. 10s. $4\frac{1}{2}$ d. *fac.*

	Ells. *s.* *d.*
	370 at 14—$2\frac{3}{4}$
	148
	37
14 s	5180 *d.*
$\frac{1}{6}$	61——8
$\frac{1}{4}$	15——5
$\frac{1}{2}$	7——$8\frac{1}{2}$
$\frac{1}{20}$	526|4——$9\frac{1}{2}$
	263 l. 4 s. $9\frac{1}{2}$ d. *facit.*

	Ells. *s.* *d.*
	136 at 9—$2\frac{1}{2}$
9 s	1224——0 d.
$\frac{1}{6}$	22——8
$\frac{1}{4}$	5——8
$\frac{1}{20}$	125|2—4
	62 l. 12 s. 4d. *facit.*

	Ells. *s.* *d.*
	431 at 2—$4\frac{1}{2}$
2 s	862
$\frac{1}{4}$	107——9 d.
$\frac{1}{2}$	53——$10\frac{1}{2}$
$\frac{1}{20}$	102|3——$7\frac{1}{2}$
	51 l. 3 s. $7\frac{1}{2}$ d. *facit.*

19. *Caſe* 6. When the given value of the integer is pounds, then multiply the number of integers whoſe value is ſought, by the price of the integer, and the product is the anſwer in pounds.

Examples.

C. *l.*
42 at 2 *per* C.
84 l. *facit.*

C. *l.*
13 at 8 *per* C.
104 l. *facit.*

C. *l.*
30 at 3 *per* C.
90 l. *facit.*

C. *l.*
48 at 12 *per* C.
576 l. *facit.*

20.

20. *Caſe* 7. If the price of the integer is pounds and ſhillings, then for the pounds work as in the laſt rule, and for the ſhillings as in the 12th and 13th rules foregoing; then add the numbers produced from them both, and the ſum is the value ſought.

Examples.

	C. *l.* *s.*
	46 at 2—4
2 l.	92 *s.*
4 s.	9—4
	101 l. 4 s. *facit.*

	Groſs. *l.* *s.*
	82 at 4—10
4 l.	328
10 s.	41
	369 l. *facit.*

	Groſs. *l.* *s.*
	58 at 3—7
3 l.	174 *s.*
6 s.	17—8
1 s.	2—18
	194 l. 6 s. *facit.*

	Groſs. *l.* *s.*
	26 at 3—15
3 l.	78
14 s.	18—4
	1—6
	97 l. 10 s. *facit.*

21. When the given price of an integer conſiſts of pounds, ſhillings, pence, and farthings, then firſt work for the ſhillings, pence, and farthings, according to the 18th rule of this chapter; and find the total value of the given number, as if there were no pounds; then work with the pounds according the 19th rule of this chapter; and add the numbers thus found, and their ſum is the total value required.

Examples of this rule follow.

	C. *l.* *s.* *d.*
	213 at 1—13—4½
	639
	213
13 s.	2769——*d.*
3 d.	53——3
1½ d.	26——7½
	284|8—10½
	142 l. 8 s. 10½ d.
1 l.	213
	355 l. 8 s. 10½ d. *fac.*

C. *l.* *s.* *d.*	
37 at 3—8—10½	
296—*d.*	8 s.
18—6	6 d.
9—3	3 d.
4—7½	1½ d.
32|8—4½	
16 l. 8 s. 4½ d.	
111	3 l.
127 l. 8 s. 4½ d. *facit.*	

	Groſs. l. s. d.
	416 at 2—9—3¾
9 s.	3744
3 d.	104
¾ d.	26
	387|4
	193 l. 14 s.
2 l.	832
	1025 l. 14 s. *facit.*

Groſs. l. s. d.	
48 at 3—15—11½	
240	
48	
720	15 s.
24	6 d.
16	4 d.
6	1½ d.
76|6—	
38 —6	
144	3 l.
182 l. 6 s. *facit.*	

22. When there is given the value of an integer, and it is required to know the value of many ſuch integers, together with ¼ or ½ or ¾ of an integer; then firſt (by the former rules) find out the value of the given number of integers; and then for ¼ of an integer take ¼ of the given value of the integer; or for ½ take ½ of the given value of the integer; and for ¾, firſt take ½ of the given value, and then ½ of that ½, ſetting each part under the precedent: then adding them together, their ſum will be the required value of the integers and their parts. *Example.* What is the value of 116½ yards at 4 s. 6 d. *per* yard? To give an anſwer, firſt I work for the value of 116 yards, by the 15th rule foregoing; and then for the ½ yard I take ½ of 4 s. 6 d. which is 2 s. 3 d. and add to the reſt found as before, then is that ſum the total value of 116½ yards at 4 s. 6 d. *per* yard; which I find to amount to 26 l. 4 s. 3 d. as by the work in the margin.

Yds. s. d.	
116½ at 4——6	
11 l. 12 s.	2 s.
14 l. 10 s.	2 s. 6 d.
2 s. 3 d.	½ yd.
26 l. 4 s. 3 d. *facit.*	

Other examples follow.

324¼ yards at 4 s. 10 d.

1296	4 s.
162	6 d.
108	4 d.
1——2½ d.	¼ yd.

156|7 s. 2½ d.

78 l. 7 s. 2½ d. *facit.*

720½ yds at 6 s. 8 d.

240 l. 3 s. 4 d. *facit.*

228¾ ells at 12 s. 11 d.

2736	12 s.
76	4 d.
76	4 d.
57	3 d.
6——5½ d.	½ ell.
3——2¾ d.	¼ ell.

295|4 s. 8¼ d.

147 l. 14 s. 8¼ d. *facit.*

C. qrs. lb. *l. s. C.*
28—3—14 at 1 10 *per*

28 l.	1 l.
14	10 s.
0—15 s.	½ C.
7 6 d.	¼ C.
3 9	14 lb.

43 l. 6 s. 3 d. *facit.*

Many more queſtions might be ſtated, and ſeveral other rules of practice might be ſhewn, according to the method of divers authors; but what have been delivered here, are ſufficient for the practical arithmetician in all caſes whatſoever.

CHAP. XXVII.

The Rule of Barter.

1. BArter is a rule among merchants, which (in the exchanging of one commodity for another) informs them ſo to proportion their rates, as that neither may ſuſtain loſs.

2. To reſolve queſtions in barter, it will not be difficult to him that is acquainted with the golden rule, or rule of three, it being altogether uſed in reſolving ſuch queſtions.

Queſt. 1. Two merchants (*viz.* A and B) barter. A hath 13 C. 3 qrs. 14 lb. of pepper, at 2 l. 16 s. *per* C. and B hath cotton at 9 d. *per* lb. I demand how much cotton B muſt give A for his pepper? *Anſ.* 9 C. 1 qr.

Firſt find by the rule of three, or the rules of practice, foregoing, how much the pepper is worth; ſaying,

If 1 C. coſt 2 l. 16 s. what will 13 C. 3 qrs. 14 lb. coſt? *Anſ.* 38 l. 17 s.

Secondly, By the rule of three, ſay, If 9 d. buy 1 lb. of cotton, how much will 38 l. 17 s. buy? *Anſ.* $9\frac{1}{4}$ C.; and ſo much cotton muſt B give to A for 13 C. 3 qrs. 14 lb. of pepper, at 2 l. 16 s. *per* C. when the cotton is worth 9 d. *per* lb.

Queſt. 2. Two merchants (A and B) barter. A hath ginger worth 1 l. 17 s. 4 d. *per* C. but in barter he will have 2 l. 16 s. *per* C. B hath nutmegs worth 5 l. 12 s. *per* C.; now I demand how B muſt rate his nutmegs *per* C. to make his gain in barter equal to that of A? *Anſ.* 8 l. 8 s.

Say, by the rule of three, if 1 l. 17 s. 4 d. require 2 l. 16 s. in the barter, what will 5 l. 12 s. require in barter? *Facit* 8 l. 8 s.

Queſt. 3. A and B barter. A hath 120 yards of broad cloth, worth 6 s. *per* yard, but in barter he will have 8 s. *per* yard. B hath ſhalloon worth 4 s. *per* yard; now I demand how many yards of ſhalloon B muſt give A for his broad-cloth, making his gain in barter equal to that of A? *Anſ.* 180 yards of ſhalloon.

Firſt (as in the laſt queſtion) find out how B ought to ſell his ſhalloon in barter, *viz.* ſay, If 6 s. require 8 s. what will 4 s. require? *Anſ.* 5 s. 4 d.

Thus you ſee that B muſt ſell his ſhalloon in barter at 5 s. 4 d. if A ſell his broad-cloth at 8 s. *per* yard.

It remaineth now to find out how much ſhalloon B muſt give for 120 yards of broad-cloth; which, after the ſame method uſed to reſolve the firſt queſtion of this chapter, is found to be 180; and ſo many yards of ſhalloon muſt B give A for the 120 yards of broad-cloth.

Queſt. 4. A and B bartered. A had 14 C. of ſugar worth 6 d. *per* lb. for which B gave him 1 C. 3 qrs of cinnamon; I demand how B rated his cinnamon *per* lb.? *Anſ.* 4 s. *per* lb.

Queſt. 5. A and B barter. A hath 4 tuns of brandy, worth 37 l. 16 s. *per* tun in ready money; but in barter he hath 50 l. 8 s. *per* tun; and A giveth B 21 C. 2 qrs. $11\frac{1}{5}$ lb. ginger for the 4 tuns of brandy; I deſire to know how much B ſold his ginger in barter *per* C. and how much it was worth in ready money? *Anſ.* For 9 l. 6 s. 8 d. in barter, and it is worth 7 l. *per* C. in ready money.

Queſt. 6. A and B barter. A hath 320 dozen of candles, at 4 s. 6 d. *per* dozen; for which B giveth him 30 l. in money, and the reſt in cotton at 8 d. *per* lb. I demand how much cotton he muſt give him more than the 30 l.? *Anſ.* 11 C. 1 qr.

Queſt. 7. A and B barter. A hath 608 yards of broad-cloth, worth 14 s. *per* yard, for which B giveth him 125 l. 12 s. ready money, and 85 C. 2 qrs. 24 lb. of bees-wax; now I deſire to know how he reckoned his wax *per* C.? *Anſ.* 3 l. 10 s. *per* C.

CHAP. XXVIII.

Queſtions in Loſs and Gain.

Queſt. 1. A Merchant bought 436 yards of broad-cloth for 8 s. 6 d. *per* yard, and ſelleth it again at 10 s. 4 d. *per* yard; now I deſire to know how much he gained in the ſale of the 436 yards? *Anſ.* 39 l. 19 s. 4 d.

Firſt, find out by the rule of three, or by practice, how much the cloth coſt him at 8 s. 6 d. *per* yard, which I find to be 185 l. 6 s.; then by the ſame rule find out how much he ſold it for, *viz* 225 l. 5 s. 4 d.; then ſubtract 185 l. 6 s. which it coſt him, from 225 l. 5 s. 4 d. which he ſold it for, and there remaineth 39 l. 19 s. 4 d. for his gain in the ſale thereof.

Otherwiſe it may ſooner be reſolved thus: Firſt, find out how much he gained *per* yard, *viz.* ſubtract 8 s. 6 d. which he gave *per* yard, from 10 s. 4 d. which he ſold it for *per* yard, the remainder is 1 s. 10 d. for his gain *per* yard. Then ſay,

If 1 yard gain 1 s. 10 d. what will 436 yards gain? The anſwer by practice, or the rule of three, is 39 l. 19 s. 4 d. as was found before.

Queſt. 2. A draper bought 124 yards of holland cloth, for

for which he gave 31 l. I deſire to know how he muſt ſell it *per* yard, to gain 10 l. 6 s. 8 d. in the whole ſale of the 124 yards? *Anſ.* At 6 s. 8 d. *per* yard.

Add the price which it coſt him (*viz.* 31 l.) to his intended gain, (*viz.* 10 l. 6 s. 8 d.) the ſum is 41 l. 6 s. 8 d. Then ſay,

If 124 yards require 41 l. 6 s. 8 d. what will 1 yard require? By the rule of three, I find the anſwer 6 s. 8 d.

Queſt. 3. A grocer bought 3 C. 1 qr. 14 lb. of cloves, which coſt him 2 s. 4 d. *per* lb. and ſold them for 52 l. 14 s. I deſire to know how much he gained in the whole? *Anſ.* 8 l. 12 s.

Queſt. 4. A draper bought 86 kerſeys for 129 l. I demand how he muſt ſell them *per* piece to gain 15 l. in laying out 100 l. at that rate? *Anſ.* 1 l. 14 s. 6 d. *per* piece. For

As 100 is to 115 l. ſo is 129 l. to 148 l. 7 s.

So that, by the proportion above, I have found how much he muſt receive for the 86 kerſeys to gain after the rate of 15 l. *per* C. Then to find how he muſt ſell them *per* piece, I ſay,

As 86 pieces are to 148 l. 7 s. ſo is 1 piece to 1 l. 14 s. 6 d. which is the number ſought.

Queſt. 5. A grocer bought $4\frac{1}{4}$ C. of pepper for 15 l. 17 s. 4 d. and (it proving to be damnified) he is willing to loſe 12 l. 10 s. *per cent.* I demand how he muſt ſell it *per* lb.? *Anſ.* 7 d. *per* lb.

Subtract 12 l. 10 s. the loſs of 100 l. from 100 l. and there remains 87 l. 10 s. Then ſay,

As 100 l. is to 87 l. 10 s. ſo is 15 l. 17 s. 4 d. to 13 l. 17 s. 8 d. ſo much as he muſt ſell it all for, to loſe after the rate propounded. Then to know how he muſt ſell it *per* lb. I ſay,

As $4\frac{1}{4}$ C. is to 13 l. 17 s. 8 d. ſo is 1 lb. to 7 d.

Queſt. 6. A plummer ſold 10 fodder of lead (the fodder containing $19\frac{1}{2}$ C.) for 204 lb. 15 s. and gained after the rate of 12 l. 10 s. *per* 100 l. I demand how much it coſt him *per* C.? *Anſ.* 18 s. 8 d.

To reſolve this queſtion, add 12 l. 10 s. (the gain *per cent.*) to 100 l. and it makes 112 l. 10 s.; then ſay,

As 112 l. 10 s. is to 100 l. ſo is 204 l. 15 s. to 182 l.

Which 182 l. is the ſum it coſt him in all. Then reduce your 10 fodders to half-hundreds, and it makes 390. Then ſay,

As 390 half-hundreds is to 182 l. ſo is 2 half-hundreds to 18 s. 8 d. the price of 2 half-hundreds, or one C. weight, and ſo much it ſtood him *per* C. weight.

Queſt. 7. A merchant bought 8 tuns of wine, which being ſophiſticated, he ſelleth for 400 l. and loſeth after the rate of 12 l. in receiving 100 l.: now I demand how much it coſt him *per* tun? and how he ſelleth it *per* gallon to loſe after the ſaid rate? *Anſ*. It coſt 56 l. *per* tun, and he muſt ſell it at 3 s. 11 d. $2\frac{10}{21}$ qrs. *per* gallon to loſe 12 l. in receiving 100 l.

To reſolve this queſtion, I conſider, in the firſt place, that in receiving 100 l. he loſeth 12 l. therefore 100 l. comes in for 112 l. laid out; wherefore to find out how much he laid out for the whole, I ſay,

As 100 l. is to 112 l. ſo is 400 l. to 448 l. and ſo much the 8 tuns coſt him. Then, to find how much it coſt *per* tun, I ſay,

As 8 is to 448 l. ſo is 1 to 56 l. the price it coſt *per* tun.

Now, to find how he muſt ſell it *per* gallon, reduce the 8 tuns into gallons, they make 2016: then ſay,

As 2016 gallons is to 400 l. ſo is 1 gallon to 3 s. 11 d. $2\frac{10}{21}$ qrs. the price he muſt ſell it at *per* gallon to loſe as aforeſaid.

Queſt. 8. A merchant bought 8 tuns of wine, which being ſophiſticated, he is willing to ſell for 400 l. and loſeth at that rate 12 l. in laying out 100 l. upon the ſame; now I demand how much it coſt him *per* tun?

Here I conſider, that for 100 l. laid out he receiveth but 88 l.; wherefore to find what the 8 tuns coſt him, I ſay,

As 88 l. is to 100 l. ſo is 400 l. to $454\frac{6}{11}$ the price it all coſt him. Then to find how much *per* tun, I ſay,

As 8 is to $454\frac{6}{11}$, ſo is 1 to $56\frac{9}{11}$, or 56 l. 16 s. 4 d. $2\frac{5}{11}$ qrs. *per* tun.

C H A P. XXIX.

Equation of Payments.

1. EQuation of payments is that rule amongſt merchants, whereby we reduce the times for pay-

ments of ſeveral ſums of money to an equated time for payment of the whole debt, without damage to debtor or creditor. And the rule is,

2. Multiply the ſums of each particular payment by its reſpective time; then add the ſeveral products together, and their ſum divide by the total debt; and the quotient thence ariſing is the equated time for the payment of the whole debt. *Example:*

Queſt. 1. A is indebted to B in the ſum of 130 l. whereof 50 l. is to be paid at 2 months, and 50 l. at 4 months, and the reſt at 6 months; now they agree to make one payment of the total ſum; the queſtion is, What is the equated time for payment, without damage to debtor or creditor?

To reſolve this queſtion, I multiply each payment by its time, *viz.*

50 l. multiplied by 2 months, produceth —— 100
50 l. multiplied by 4 months, produceth —— 200
30 l. multiplied by 6 months, produceth —— 180

The ſum of the products is —— —— 480

Then I divide 480 (the ſum of the product) by 130 (the total debt), and the quotient is $3\frac{9}{13}$ months, for the time of paying that whole debt.

Queſt. 2. A merchant hath owing him 1000 l. to be paid as followeth, *viz.* 600 l. at 4 months, 200 l. at 6 months, and the reſt (which is 200 l.) at 12 months, and he agreeth with his debtor to make one payment of the whole: I demand the time of payment without damage to debtor or creditor?

600 l. multiplied by 4 months, is — — 2400
200 l. multiplied by 6 months, is — — 1200
200 l. multiplied by 12 months, is — — 2400

The ſum of the products is — — — 6000

And the ſum of the products (6000) being divided by the whole debt (1000 l.), quotes 6 months for the time of payment of the whole debt.

The proof of the rule of equation of payments.

3. The truth of this rule is thus manifeſt, if the intereſt of that money which is paid (by the equated time) after it is due, be equal to the intereſt of that money, which (by the equated time) is paid ſo much ſooner than it is due at any rate *per* C. then the operation is true, otherwiſe not. For example:

In the laſt queſtion 600 l. ſhould have been paid at 4 months, but is not diſcharged till 6 months, (that is, 2 months after it is due); wherefore its intereſt for 2 months at 6 *per cent. per ann.* is 6 l.; and then 200 l. was to be paid at 6 months, which is the equated time for its payment, therefore no intereſt is reckoned for it: but 200 l. ſhould have been paid at 12 months, but is paid at 6 months, which is 6 months ſooner than it ought; wherefore the intereſt of 200 l. for 6 months, is 6 l. (accounting 6 l. *per cent. per ann.*), which is equal to the intereſt of 600 l. for 2 months: wherefore the work is right.

Queſt. 3. A merchant hath owing him a certain ſum to be diſcharged at three equal payments, *viz.* $\frac{1}{3}$ at two months, $\frac{1}{3}$ at four months, and $\frac{1}{3}$ at eight months; the queſtion is, What is the equated time for the payment of the whole debt?

In queſtions of this nature (*viz.* where the debt is divided into equal or unequal parts) each of the parts is to be multiplied by its time, and the ſum of the products is the anſwer.

$\frac{1}{3}$ multiplied by 2 months, produceth $\frac{2}{3}$
$\frac{1}{3}$ multiplied by 4 months, produceth $1\frac{1}{3}$
$\frac{1}{3}$ multiplied by 8 months, produceth $2\frac{2}{3}$

The ſum of the products is — $4\frac{2}{3}$

which is $4\frac{2}{3}$ months for the equated time of payment.

If inſtead of the fractions repreſenting the parts, you had wrought by the numbers themſelves (repreſented by thoſe parts), according to the firſt and ſecond example, it would have been the ſame anſwer; and ſuppoſe the debt had been 90 l. then $\frac{1}{3}$ of it is 30 l. for each payment, *viz.* at 2, 4, and 8 months. Then

30 l. multiplied by 2 months, produceth 60
30 l. multiplied by 4 months, produceth 120
30 l. multiplied by 8 months, produceth 240

The ſum of the products is — 420

which divided by 90 (the whole debt), quoteth $4\frac{60}{90}$, or $4\frac{2}{3}$ months, as before.

Queſt. 4. A merchant oweth a ſum of money to be paid $\frac{1}{2}$ at 5 months, and $\frac{1}{4}$ at 8 months, and $\frac{1}{4}$ at 10 months, and he agreeth with his creditor to make one total payment; I demand the time without damage to debtor or creditor? Work as in the laſt queſtion, and you will find the anſwer to be 7 months.

Queſt. 5. A is indebted to B 640 l. whereof he is to pay 40 l. preſent money, 350 l. at 3 months, and the reſt (*viz.* 250 l.) at 8 months, and they agree to make an equated time for the whole payment; now I demand the time?

In queſtions of this nature (*viz.* where there is ready money paid) you are in multiplying to neglect the money that is to be paid preſent, and work with the reſt, as is before directed; and divide the ſum of the products by the whole debt, and the quote is the anſwer: for here 40 l. is to be paid preſent, and hath no time allowed, and, according to the rule, it ſhould be multiplied by its time, which is (0); therefore 40 times 0 is 0, which neither augmenteth nor diminiſheth the dividend: wherefore to proceed (according to direction), I ſay,

350 by 3 months, produceth — — 1050
250 by 8 months, produceth — — 2000

The ſum of the products is —— 3050

which divided by 640, the whole debt, the quote is $4\frac{49}{64}$ months, the time of payment.

Queſt. 6. A is indebted to B in a certain ſum, half whereof is to be paid preſent money, one third at 6 months, and the reſt at 8 months; now I demand the equated time for payment of it all? *Anſ.* $3\frac{1}{3}$ months is the time of payment.

Queſt. 7. A is indebted to B 120 l. whereof $\frac{1}{3}$ is to be paid at 3 months, $\frac{1}{4}$ at 6 months, and the reſt at 9 months; what is the equated time for the payment of the whole ſum? *Anſ.* At 6$\frac{1}{4}$ months.

Queſt. 8. A is indebted to B 420 l. which is due at the end of 6 months, but A is willing to pay him 140 l. preſent, provided he can have the remainder forborn ſo much the longer to make ſatisfaction for his kindneſs; which is agreed upon: I deſire to know what time ought to be allotted for the payment of the 280 l. remaining?

To reſolve this queſtion, firſt, find out what is the intereſt of 140 l. for the time it was paid before it was due at 6 *per cent.* or any other rate, (*viz.* 6 months), and you will find it to be 4 l. 4 s. Then it is evident, that the remaining 280 l. muſt be detained ſo much longer than 6 months, as the while it may eat out that intereſt, *viz.* 4 l. 4 s.; which is thus found out, *viz.* Firſt, ſee what is the intereſt of 280 l. for a month, or any other time; but here we will take one month, and its intereſt for one month is 28 s.

Then, by the rule of three, ſay,

As 28 s. is to 1 month, ſo is 84 s. to 3 months; ſo that the 280 l. remaining muſt be kept 3 months beyond its firſt time of payment, (*viz.* 6 months); which added thereto, makes 9 months; at the end of which time A ought to make payment of the remainder.

CHAP. XXX.

Exchange.

1. THE rule of exchange informeth merchants how to exchange monies, weights, or meaſures of one country into (or for) the monies, weights, or meaſures of another country; and when the rate, reaſon, or proportion betwixt the money, weights, or meaſures of different countries is known, it will not be difficult for the practitioner, that is well acquainted with the rule of proportion, (or rule of three), to reſolve any queſtion, wherein it is required to exchange a given quantity of the one kind into the ſame value of another kind.

2. In queſtions of exchange there is always a compariſon made between the coins, *&c.* of two countries (or kinds) or more.

3. In queſtions where there is a compariſon made between two things (whether they be monies, weights, *&c.*) of different kinds (or countries), there may be a ſolution found by a ſingle rule of three; as may appear by the following example.

Queſt. 1. A merchant at London delivered 370 l. Sterling, to receive the ſame at Paris in French crowns, the exchange $3\frac{1}{3}$ French crowns *per* pound Sterling; I demand how many French crowns he ought to receive?

In placing the numbers, obſerve the 6th rule of the 10th chapter; which being done, the given numbers will ſtand as in the margin; and being reduced according to the rules of the 24th chapter, will ſtand thus,

l. *crowns.* *l.*
1 : $3\frac{1}{3}$ :: 370

As $\frac{1}{1}$ is to $\frac{10}{3}$, ſo is $\frac{370}{1}$ to $1233\frac{1}{3}$.

So that I conclude he ought to receive $1233\frac{1}{3}$ French crowns at Paris for his 370 l. delivered at London.

Queſt. 2. A merchant delivered at Amſterdam 587 l. Flemiſh, to receive the value thereof at Naples in ducats, the exchange $4\frac{4}{5}$ ducats *per* l. Flemiſh; I demand how many ducats he ought to receive?

The proportion is as followeth.

l. *Ducats.* *l.* *Ducats.*
As 1 is to $\frac{24}{5}$, ſo is $\frac{587}{1}$ to $2817\frac{3}{5}$.

So I find he ought to receive $2817\frac{3}{5}$ ducats at Naples for the 587 l. Flemiſh delivered at Amſterdam.

Queſt. 3. A merchant at Florence delivereth 3478 ducatoons, to receive the value at London in pence, the exchange at $53\frac{1}{2}$ pence Sterling *per* ducatoon; I demand how much Sterling he ought to receive?

The proportion for reſolution is,

Ducat. *d.* *Ducats.* *d.*
As $\frac{1}{1}$ is to $\frac{107}{2}$, ſo is $\frac{3478}{1}$ to 186073.

which is equal to 775 l. 6 s. 1 d. for the anſwer.

I might here (according to the cuſtom of arithmetical writers) lay down tables for the reduction of foreign coins into Engliſh; but by reaſon of their inſtability (for they continue not at a conſtant ſtandard, as our Sterling money doth, but are ſometimes raiſed and ſometimes depreſſed) I ſhall forbear.

4. When there is a compariſon made between more than two different coins, weights, or meaſures, there ariſe ordinarily two different caſes from ſuch a compariſon.

1. When it is required to know how many pieces of the firſt coin, weight, or meaſure, are equal in value to a known number of pieces of the laſt coin, weight, or meaſure.

2. When it is required to find out how many pieces of the laſt coin, weight, or meaſure, are equal in value to a given number of the firſt ſort of coin, weight, or meaſure.

An example of the firſt caſe may be this, viz.

Queſt. 4. If 150 pence at London are equal to 3 ducats at Naples, and $4\frac{4}{5}$ ducats at Naples make $34\frac{1}{2}$ ſhillings at Bruſſels; then how many pence at London are equal to 138 ſhillings at Bruſſels? *Facit* 960 d.

This queſtion may be reſolved by two ſingle rules of three: for, firſt, I ſay,

If $\frac{3}{1}$ ducats at Naples make 150 d. at London, how many pence will $4\frac{4}{5}$ ducats make? *Anſ.* 240 d.

By the foregoing proportion, we have diſcovered that $4\frac{4}{5}$ ducats at Naples make 240 pence at London: and by the tenor of the queſtion, we ſee that $4\frac{4}{5}$ ducats at Venice make $34\frac{1}{2}$ ſhillings at Bruſſels; therefore 240 d. at London are equal to $34\frac{1}{2}$ ſhillings at Bruſſels, (for the things that are equal to one and the ſame thing, are alſo equal to one another); wherefore we have a way laid open to give a ſolution to this queſtion by another ſingle rule of three, whoſe proportion is,

As $34\frac{1}{2}$ ſhillings at Bruſſels is to 240 pence at London, ſo is 138 ſhillings at Bruſſels to 960 pence at London; which is the anſwer to the queſtion.

An example of the ſecond caſe may be this, viz.

Queſt. 5. If 40 lb. Avoirdupois weight at London is equal to 36 lb. weight at Amſterdam, and 90 lb. at Amſterdam makes 116 lb. at Dantzic; then how many pounds at Dantzic are equal to 112 lb. Avoirdupois weight at London? *Anſ* $129\frac{23}{25}$ lb. at Dantzic.

This queſtion is likewiſe anſwered by two ſingle rules of three, *viz.* Firſt I ſay,

As 36 lb. at Amſterdam is to 40 lb. at London,
So is 90 lb. at Amſterdam to 100 lb. at London.

And by the queſtion you find, that 90 lb. at Amſterdam is 116 lb. at Dantzic; and therefore 100 lb. at London is likewiſe equal thereunto; wherefore again I ſay,

As 100 lb. at London is to 116 lb. at Dantzic,
So is 112 lb. at London to $129\frac{23}{25}$ lb. at Dantzic.

By which I find that $129\frac{23}{25}$ lb. at Dantzic are equal to 112 lb. Avoirdupois weight at London.

5. There is a more ſpeedy way to reſolve ſuch queſtions as are contained under the two caſes before mentioned, laid down by Mr Kerſey in the third chapter of his appendix to Wingate's arithmetic, where he hath given two rules for the reſolution of the queſtions pertinent to the two ſaid caſes.

6. But I ſhall lay down a general rule for the ſolution of both caſes. And firſt, Let the learner obſerve the following directions in placing of the given terms, *viz.*

7. Let there be made two columns, and in theſe columns ſo place the given terms, one over the other, as that in the ſame column there may not be found two terms of the ſame kind, one with the other.

Having thus placed the terms, the general rule is,

Obſerve which of the ſaid columns hath the moſt terms placed in it, and multiply all the terms therein continually, and place the laſt product for a dividend; then multiply the terms in the other column continually, and let the laſt product be a diviſor; then divide the ſaid dividend by the ſaid diviſor, and the quotient thence ariſing is the anſwer to the queſtion.

So the example of the firſt of the ſaid caſes being again repeated, *viz.* If 150 pence at London make three ducats at Naples, and $4\frac{4}{5}$ ducats at Naples make $34\frac{1}{2}$ ſhillings at Bruſſels; then how many pence at London are equal to 138 ſhillings at Bruſſels?

The terms being placed according to the 7th rule, will ſtand as followeth.

	A	B	
Pence at London	150	3	Ducats at Naples.
Ducats at Naples	$4\frac{4}{5}$	$34\frac{1}{2}$	Shillings at Bruſſels.
Shillings at Bruſſels	138		

Having thus placed the terms, that in neither column there are two terms of one kind, then obſerve, that the column under A hath moſt terms in it; therefore they muſt be multiplied together for a dividend, *viz.* 150 multiplied by $4\frac{4}{5}$, produceth $\frac{3600}{5}$; which multiplied by 138, produceth $\frac{496800}{5}$ for a dividend; then in the column under B there are 3, and $34\frac{1}{2}$, which multiplied together, produce $\frac{207}{2}$ for a diviſor; then having divided $\frac{496800}{5}$ by $\frac{207}{2}$, the quotient is 960 pence for the anſwer to the queſtion.

Again, Let the example of the ſecond caſe be again repeated, *viz.* If 40 lb. Avoirdupois weight at London make 36 lb. weight at Amſterdam, and 90 lb. at Amſterdam make 116 lb. at Dantzic; then how many pounds at Dantzic are equal to 112 lb. Avoirdupois weight at London?

The terms being diſpoſed according to the 7th rule foregoing, will ſtand thus:

	A	B	
lb. at London	40	36	lb. at Amſterdam
lb. at Amſterdam	90	116	lb. at Dantzic
		112	lb. at London.

Whereby I find that the terms under B multiplied together, produce 467712 for a dividend; and the terms under A, *viz.* 40 and 90, produce 3600 for a diviſor; and diviſion being finiſhed, the quotient giveth $129\frac{3312}{3600}$ or $129\frac{23}{25}$ pounds Dantzic for the anſwer.

C H A P. XXXI.

Single Poſition

1. NEgative arithmetic, called *the rule of falſe*, is that by which we find ~~out~~ a truth, by numbers invented or ſuppoſed. And this is either ſingle or double.

2. The rule of ſingle poſition, is, when at once, *viz.*

by one falſe poſition, or feigned number, we find out the true number ſought.

3. In the ſingle rule of falſe, when you have made choice of your poſition, work it according to the tenor of the queſtion, as if it were the true number ſought; and if by the ordering your poſition you find either the reſult too much or too little, you may then find out the number ſought by the proportion following, *viz.*

As the reſult of your poſition is to the poſition, ſo is the given number to the number ſought. For example:

Queſt. 1. A perſon having about him a certain number of crowns, ſaid, If a fourth, and third, and ſixth of them were added together, they would make juſt 45 crowns; now I demand the number of crowns he had about him? *Anſ.* 60 crowns.

To reſolve this queſtion, I ſuppoſe he had 24 crowns, (or any other number that will admit of the like diviſion); now the fourth of 24 is 6, and the third is 8, and the ſixth is 4; all which parts (*viz.* 6, 8, and 4) being added together, make but 18, but it ſhould be 45. Wherefore I ſay, by the rule of three,

As 18, the ſum of the parts, is to the poſition 24, ſo is 45, the given number, to 60, the true number ſought.

For the fourth of 60 is 15, and the third of 60 is 20, and the ſixth of 60 is 10; which added together make 45.

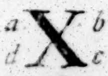
Queſt. 2. Three perſons, *viz.* A, B, C, thus diſcourſe together concerning their age. Quoth B to A, I am as old, and half as old again as you; then quoth C to B, I am twice as old as you; then quoth A to them, and I am ſure the ſum of all our ages is 165. Now I demand each man's age? *Anſ.* A 30, B 45, C 90 years of age; which added together, make 165.

CHAP. XXXII.

Double Poſition.

1. THE rule of double poſition is, when two falſe poſitions are aſſumed to give a reſolution to the queſtion propounded.

2. When any queſtion is ſtated in double poſition, make ſuch a croſs as in the margin.



3. Then make choice of any number you think may be convenient for your working, which call your firſt poſition, and place it at the end of the croſs at *a*; then work with this poſition as if it were the true number ſought, according to the nature of your queſtion; then having found out your error, either too much or too little, place it on the ſide of the croſs at *d*; then make choice of another number of the ſame denomination with the firſt poſition, (which call your ſecond poſition), and place it on the other ſide of the croſs at *b*; then work with this poſition as with the former; and having found out your error, either too much or too little, place it on the ſide of the croſs at *c*, and then the poſitions will ſtand at the top of the croſs, and the errors at the bottom, each under its correſpondent poſition; and then multiply the errors into the poſitions croſſwiſe; that is to ſay, multiply the firſt poſition by the ſecond error, and the ſecond poſition by the firſt error, and put each product over its poſition.

4. Having proceeded ſo far, then conſider whether the errors are both alike; that is, whether they are both too much or both too little; and if they are alike, then ſubtract the leſſer product from the greater, and ſet the remainder for a dividend; then ſubtract the leſſer error from the greater, and let the remainder be a diviſor; then the quotient ariſing by this diviſion is the anſwer to the queſtion.

5. But if the errors are unlike, that is, one too much, and the other too little, then add the products of the poſitions and errors together, and their ſum ſhall be a dividend; then add the errors together, and their ſum ſhall be a diviſor; and the quotient ariſing thence, is the anſwer. Which two laſt rules may be kept in memory by the verſe following, *viz.*

When errors are of unlike kinds,
Addition doth enſue;
But if alike, ſubtraction finds
Dividing work for you.

Queſt. 1. A, B, and C, built a houſe which coſt 76l. of which A paid a certain ſum unknown, B paid as much as A, and 10l. over, and C paid as much as A and B; now I deſire to know each man's ſhare in that charge?

Having made a croſs according to the ſecond rule, I come according to the third rule, to make choice of my firſt poſition. And here I ſuppoſe A paid 6 l. which I put upon the croſs, as you ſee; then B paid 16 l. for it is ſaid he had paid 10 l. more than A; and C paid 22 l. for it is ſaid he paid as much as A and B: then I add their parts, and they amount to 44. But it is ſaid they paid 76 l. wherefore there is 32 too little, which I note down at the bottom of the croſs under its poſition, for the firſt error.

	l.					*l.*
A	6				A	9
B	16				B	19
C	22	120	168	288	C	28
Sum	44	6	X	9	Sum	56
	76	12)		(14		76
	44	32		20		56
Error	32		12		Error	20

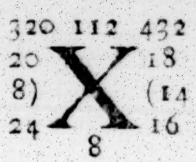
Secondly, I ſuppoſe A paid 9 l. then B paid 19 l. and C 28 l.; all which added together, makes 56. But they ſhould make 76; wherefore the error of this poſition is 20; which I put at the bottom of the croſs under its poſition, for the ſecond error. Then I multiply the errors and poſitions croſſwiſe, *viz.* 32 (the error of the firſt poſition) by 9 (the ſecond poſition), and the product is 228; then I multiply 20 (the error of the ſecond poſition) by 6 (the firſt poſition), and the product is 120; which products I place on the top of the croſs, over their reſpective poſitions.

Then (according to the 4th rule) I ſubtract the leſſer product from the greater, *viz.* 120 from 288, becauſe the errors are both alike, (*viz.* too little), and there remaineth 168 for a dividend; then I ſubtract 20 (the leſſer error) from 32 (the greater error), and the remainder is 12 for a diviſor; then I divide 168 by 12, and the quotient is 14 for the anſwer; which is the ſhare of A in the payment.

6. Again, ſecondly, If the errors had been both too big, it had had the ſame effect; as appeareth by the following work. For, firſt, I ſuppoſe A paid 20 l. then B paid 3 l. and C 50 l. which in all is 100 l. But it ſhould have been no more than 76; wherefore the firſt error is 24 too much. Again, I ſuppoſe A paid 18 l. then B muſt pay 28 l. and C muſt pay 46 l. which in all is 92 l. But it ſhould have been but 76 l.; wherefore the ſecond error is 16 too

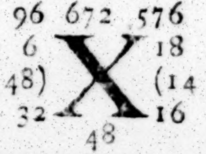
much. Then I multiply 20 (the firſt poſition) by 16 (the ſecond error), and the product is 320. Again I multiply 18 (the ſecond poſition) by 24 (the firſt error), and the product is 432. Then, becauſe the errors are both too much, I ſubtract 320 (the leſſer product) from 432 (the greater product), and there remaineth 112 for a dividend; likewiſe I ſubtract 16 (the leſſer error) from 24 (the greater error), and the difference is 8 for a diviſor; then perform diviſion, and the quotient is 14, as before, for the anſwer.

	l.
A	20
B	30
C	50
Sum	100
Subtr.	76
Error	24



	l.
A	18
B	28
C	46
Sum	92
Subtr.	76
Error	16

Again, thirdly, If the errors had been, the one too big, aed the other too little, reſpect being had to the 5th rule foregoing, the anſwer would have been the ſame. As, thus, I take for my firſt poſition 6, and then the error is 32 too little; then I take for my ſecond poſition 18, and then the error is 16 too much; then I multiply the poſitions and errors croſſwiſe, and the products are 96 and 576; and becauſe the errors are unlike, *viz.* one too big, and another too little, I add the products 96 and 576 together, and their ſum is 672 for a dividend: I likewiſe add the errors 32 and 16 together, and their ſum is 48 for a diviſor; then having finiſhed diviſion, I find the quotient to be 14; which is the anſwer, as was found out at the two ſeveral trials before.



For proof of the work, I ſay,	—	*l.*
If A paid — — — — —	—	14
Then B paid 14 and 10 (that is)	—	24
Then C paid 14 and 24 (that is)	—	38
The ſum of all is — —	—	76

which is the total value of the building, and equal to the given number.

Thoſe who deſire to ſee the demonſtration of this rule, let them read the 7th chapter of Mr Kerſey's *Appendix to Mr Wingate's arithmetic*, Petiſcus in the 5th book of his *Trigonometria*, or Mr Oughtred in his *Clavis mathematica*.

Queſt. 2. Three perſons, A, B, and C, thus diſcourſed together concerning their age. Quoth A, I am 18 years of age; quoth B, I am as old as A and ½ C; and quoth C, I am as old as you both, if your years were added together. Now I deſire to know the age of each perſon? *Anſ*. A is 18, B is 54 and C is 72 years of age.

Queſt. 3. A father lying at the point of death, left to his three ſons, *viz*. A, B, C, all his eſtate in money, and divided it as followeth, *viz*. To A he gave the half wanting 44 l. to B he gave a third and 14 l. over, and to C he gave the remainder, which was 82 l. leſs than the ſhare of B. Now I demand what was the ſum left, and each man's part? *Anſ*. The ſum bequeathed was 588 l. whereof A had 250 l. B had 210 l. and C had 128 l.

Queſt. 4. Two perſons, *viz*. A and B, had each in their hands a certain number of crowns; and A ſaid to B, If you give me one of your crowns, I ſhall have five times as many as you; and ſaid B to him again, If you give me one of yours, then we ſhall each of us have an equal number. Now I demand how many crowns each perſon had? *Anſ*. A had 4, and B had 2 crowns.

Queſt. 5. What number is that unto which if I add one fourth of itſelf, and from the ſum ſubtract one eighth of itſelf, the remainder will be 216? *Anſ*. 192.

Many more queſtions may be added; but theſe well underſtood, will be ſufficient (even to the meaneſt capacity) for the reſolution of any other queſtion pertinent to this rule.

There may be an objection made, becauſe we have not treated particularly upon intereſt and rebate; but the operation of ſuch queſtions being more applicable to decimals, they are omitted, till we come to acquaint the learner with decimal arithmetic.

Laus Deo ſoli.

FINIS

Thomas is Vagabonde